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Answer Key

Correct Answer Shown

Align Bottom and Left Edge

1. When you multiply 28 times 49, what is your first line product?

  ○ 239   ○ 89

  ○ 392   ○ 98

2. When you add electronic values of 60 watts, 34 volts, and 20 ohms, their total is _____.

  ○ 114 watts

  ○ 114 volts

  ○ 60 watts + 34 volts + 20 ohms

  ○ 114 ohms

3. When you subtract –52 from –121, what is the difference?

  ○ –173   ○ +69

  ○ ±173   ○ –69

4. What is the sum of (64 – 128 + 12 – 6) × 90?

  ○ 5220   ○ 64

  ○ –5220   ○ –64

5. What is the quotient of –1770 divided by –3 ?

  ○ +590   ○ +5310

  ○ –590   ○ –5310

6. How do we arrive at the final product in long form multiplication?

  ○ subtract inverses

  ○ add line products

  ○ multiply sums

  ○ offset subtrahends

7. In the number 63,428, the place value of the six is

  _____.

  ○ ten thousands   ○ thousands

  ○ hundred thousands   ○ zero

1. A logarithmic scale _____.
    ◯ helps find perimeters
    ◯ is a scale of addition
    ◯ is one where proportional increases occupy the
         same space
    ◯ provides trigonometric functions

2. How many significant digits are there in 43.6280 ?
    ◯ 1   ◯ 3
    ◯ 5   ◯ 2

3. When using a slide rule to multiply, which scale index
    is set over the multiplier?
    ◯ K-scale index   ◯ C-scale index
    ◯ D-scale index   ◯ A-scale index

4. When using a slide rule to divide, where is the number
    being divided located?
    ◯ D-scale   ◯ LL-scale
    ◯ C-scale   ◯ Ci-scale

5. Express 650,000 watts as a power of 10.
    ◯ 6.5×10^3 watts   ◯ 6.5×10^5 watts
    ◯ 6.5×10^4 watts   ◯ 6.5×10^6 watts

6. Which scales help find squares and square roots?
    ◯ K and L   ◯ S and T
    ◯ A and D or   ◯ C and D
      B and C

7. If 3 on the C-scale is over 6 on the D-scale, where is
    the left C index?
    ◯ 12   ◯ 2
    ◯ 18   ◯ 5

8. If the C-scale index is on 9 and the cursor on the
    C-scale is on 36, what is on the D-scale?
    ◯ 324   ◯ 4
    ◯ 2.25   ◯ .25

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1.   A normal 4-function calculator has _____.
     ○ 10 keys   ○ 16 keys
     ○ 12 keys   ○ 20 keys

2.   Before each new calculation, you should _____.
     ○ press the plus-equals button
     ○ clear the register
     ○ place the decimal
     ○ enter all the variables

3.   One method of finding a square root on a calculator
     is a(an)_____ .
          ○ subtraction process
          ○ inverse process
          ○ averaging process
          ○ geometric method

4.   A display panel shows 4039. How would you divide
     by 3 ?
     ○ [+], 3, [÷] , 4039   ○ [÷] , 3, [±]
     ○ 4039, [±] , 3, [÷]   ○ [CE] , [X] , 3

5.   Then to subtract 2300 from the result?
     ○ [±] , 2300, [−]   ○ 2300, [X] , [−] , 3
     ○ [−] , 2300, [±]   ○ [X] , 5, [−] , 2300, [±]

6.   Unlike a slide rule, most calculators_____.
          ○ can find trigonometric functions
          ○ can calculate square roots
          ○ place the decimal
          ○ can handle chain operations

7.   Electronics data is usually accurate to _____.
          ○ 6 significant figures
          ○ all figures shown
          ○ 3 significant figures
          ○ one part per million

1. What is the most error you should introduce when rounding off?
   - ◯ 1%
   - ◯ 3%
   - ◯ 5%
   - ◯ 7%

2. What is the percent error made when rounding 64.6 to 65?
   - ◯ 1%
   - ◯ 2%
   - ◯ 6%
   - ◯ .6%

3. What is the percentage error in rounding off 7836 and 2425 and adding them?
   - ◯ less than ½ of 1%
   - ◯ 2%
   - ◯ 4%
   - ◯ 5%

4. When 14.3 amps and 12.75 ohms are rounded off, what is the voltage and % error?
   - ◯ 1%
   - ◯ 3%
   - ◯ 1.7%
   - ◯ 4%

5. Which is not a correct rounding off procedure for 115.5 amps and 115.5 volts?
   - ◯ round both to 115
   - ◯ round both to 116
   - ◯ round both to 120
   - ◯ round one to 115, the other to 116

6. What is an estimated value for the resistance where there are 4.25 milliamps and 6.75 volts?
   - ◯ 30 K-ohms
   - ◯ 32 K-ohms
   - ◯ 28 K-ohms
   - ◯ 25 K-ohms

7. What is the percentage error in rounding 4.25 to 4 and 6.75 to 7?
   - ◯ 1%
   - ◯ 2%
   - ◯ 3%
   - ◯ 4%

1. What is the sum of -242 and $+83$?

  ◯ -325 ◯ $+325$

  ◯ $+159$ ◯ -159

2. What is the sum of $-12 + 22 + (-60) + 12$?

  ◯ $+106$ ◯ $+74$

  ◯ -38 ◯ -14

3. What is the difference between -130 and -65?

  ◯ -65 ◯ $+65$

  ◯ -195 ◯ $+195$

4. What is the result of $(\frac{-18 + 6}{3})(-6)$?

  ◯ -48 ◯ $+24$

  ◯ $+48$ ◯ -24

5. What is $1\frac{4}{5}$ times $-\frac{3}{8}$ divided by $\frac{9}{10}$ in lowest terms?

  ◯ $\frac{183}{320}$ ◯ $-\frac{3}{4}$

  ◯ $\frac{320}{183} = 1\frac{137}{183}$ ◯ $+\frac{15}{16}$

6. How would 6 times the quantity -8 plus -9 divided by 3 be written?

  ◯ $(6)(-8) \div (3) \times (9)$

  ◯ $\frac{6[(-8) + (-9)]}{3}$

  ◯ $(-8) \times 9 \div 3 + 6$

  ◯ $\frac{-8 - 6}{9} \times 3$

7. What is the product of $\frac{3}{x} \times -\frac{5}{y}$?

  ◯ $\frac{2}{x - y}$ ◯ $\frac{8}{x + y}$

  ◯ $-\frac{15}{xy}$ ◯ $\frac{3y}{5x}$

8. How would $\frac{11}{12}$ divided by $\frac{3}{4}$ be worked?

  ◯ $\frac{11}{12} \times \frac{4}{3} = \frac{44}{36}$ ◯ $\frac{11}{12} \times \frac{3}{4} = \frac{33}{48}$

  ◯ $\frac{3}{4} \times \frac{12}{11} = \frac{36}{44}$ ◯ $\frac{12}{11} \times \frac{4}{3} = \frac{48}{33}$

1. What is the sum of $\dfrac{6}{4B^2 + 3C^3 - 2y}$ and $\dfrac{18}{4B^2 + 3C^3 - 2y}$?

   ○ $\dfrac{12}{4B^2 + 3C^3 - 2y}$ ○ $\dfrac{24}{8B^2 + 6C^3 - 4y}$

   ○ $\dfrac{108}{16B^4 + 9C^6 - 4y^2}$ ○ $\dfrac{24}{4B^2 + 3C^3 - 2y}$

2. What is the sum of $\dfrac{2}{3} + \dfrac{4}{5} + \dfrac{8}{15}$?

   ○ $\dfrac{14}{23}$ ○ $\dfrac{15}{16}$

   ○ $\dfrac{30}{15} = 2$ ○ $\dfrac{5}{12}$

3. What is the lowest common denominator for $\dfrac{1}{9}$, $\dfrac{2}{6}$, and $\dfrac{1}{4}$?

   ○ 18   ○ 36
   ○ 24   ○ 54

4. What is the sum of $\dfrac{3}{8} + \dfrac{2}{9}$?

   ○ $\dfrac{5}{17}$ ○ $\dfrac{1}{3}$

   ○ $\dfrac{5}{12}$ ○ $\dfrac{43}{72}$

5. What is the result of $\dfrac{4}{5} + \left(-\dfrac{1}{3}\right)$?

   ○ $\dfrac{17}{15} = 1\dfrac{2}{15}$ ○ $\dfrac{3}{5}$

   ○ $\dfrac{7}{15}$ ○ $-\dfrac{7}{15}$

6. What is the quotient of 2¼ divided by $\dfrac{3}{8}$?

   ○ 6   ○ $\dfrac{1}{6}$

   ○ $\dfrac{27}{32}$ ○ $\dfrac{3}{16}$

7. What is $\dfrac{5}{8} - \dfrac{2}{3}$?

   ○ $\dfrac{31}{24} = 1\dfrac{7}{24}$ ○ $-\dfrac{1}{24}$

   ○ $\dfrac{1}{24}$ ○ $\dfrac{7}{12}$

1. What is the square of 14?
    ○ 169   ○ 256
    ○ 526   ○ 196

2. What is the $\sqrt{2}$?
    ○ 1.212   ○ 1.313
    ○ 1.414   ○ 1.515

3. How is 3,425,596 correctly rounded off to three
    numbers?
    ○ 3425×10^3 ○ 343×10^4
    ○ $34,255 \times 10^2$ ○ 342×10^4

4. If $\sqrt{64}$ = 8 and $\sqrt{81}$ = 9, what is the approximate
    value of $\sqrt{72}$?
    ○ 8.5   ○ 8.1
    ○ 8.7   ○ 9.1

5. What is $\sqrt{200}$?
    ○ 44.3   ○ 16
    ○ 20   ○ 14.14

6. What is the square of 24?
    ○ 484   ○ 625
    ○ 576   ○ 727

7. Eight is rounded off to ___ in order to find its square root.
    ○ 800×10^{-2} ○ 80×10^{-1}
    ○ 8×10^0 ○ 8×10^{-3}

8. What is $\sqrt{800}$?
    ○ 25.25   ○ 26.25
    ○ 29.25   ○ 28.25

9. What is $\sqrt{8}$?
    ○ .282   ○ 4
    ○ 2.825   ○ 28.25

1. What is 34,329 in scientific notation?
    ○ 34329   ○ 3.43×10^4
    ○ 34.329   ○ 343.29×10^2

2. What is 6×10^{-6} in decimal fraction form?
    ○ .00006   ○ .00000006
    ○ .000006   ○ .0006

3. How is 45 megohms expressed?
    ○ 45×10^6 ohms   ○ 45×10^7 ohms
    ○ 4.5×10^8 ohms   ○ 4.5×10^5 ohms

4. What is $\dfrac{4 \times 10^8}{2 \times 10^4}$?

    ○ 8×10^4 ○ 8×10^{12}
    ○ 2×10^4 ○ 2×10^{12}

5. What is $\sqrt{1.69 \times 10^2}$?
    ○ 13   ○ 1.3×10^2
    ○ 1.3   ○ 13×10^3

6. What voltage is obtained when 3 milliamps is multiplied
    by 40,000 ohms?
    ○ .012 volts   ○ 1.2 volts
    ○ 120 volts   ○ .12 volts

7. What is $\dfrac{(10^8)^2 (10^4)^3}{10^8}$?

    ○ 10^{15} ○ 10^{20}
    ○ 10^{25} ○ 10^{36}

8. What is $\dfrac{(3 \times 10^{-9})^2 (2 \times 10^8)^3}{12 \times 10^5}$

    ○ 60   ○ 72×10^{16}
    ○ 36×10^{-54} ○ 6×10^{-24}

1. Which of these is an equation?
    - ⬭ $6x > 12$
    - ⬭ $3a - 2b$
    - ⬭ $4y$
    - ⬭ $5m + 2n = 9$

2. Which is a root for $2x + 3 = 15$?
    - ⬭ 3
    - ⬭ 9
    - ⬭ 6
    - ⬭ 12

3. What is $(x^2 + 4)$ times $(-2x)$?
    - ⬭ $-2x^3 - 8x$
    - ⬭ $4x^2 - 2x$
    - ⬭ $x^2 + 2x - 8$
    - ⬭ $-x + 2$

4. A term is _____ .
    - ⬭ at least three variables which add to give 5
    - ⬭ the square root of a negative number
    - ⬭ a combination of numbers or letters by
           multiplication
    - ⬭ another name for the solution of an equation

5. What is $2(14ab - 6c + 12 - 8ab)$?
    - ⬭ $24abc$
    - ⬭ $4ab$
    - ⬭ $12ab - 12c + 24$
    - ⬭ $18ab + 6$

6. What is the product of $4x + 2y$ times $3a - 2b$?
    - ⬭ $12ax + 6ay - 8xb - 4yb$
    - ⬭ $3axyb$
    - ⬭ $3xyb - 2axz$
    - ⬭ $-12ayb + 36$

7. What is the quotient of $\frac{x^2 + 2xy + y^2}{x + y}$?
    - ⬭ x^2y^2
    - ⬭ $x - y$
    - ⬭ x^2
    - ⬭ $x + y$

8. What is $(4x - 3y)(2x - y)$?
    - ⬭ $6x^2y - 2x + 4xy$
    - ⬭ $4y^2 - 8x^2y$
    - ⬭ $8x^2 - 10xy + 3y^2$
    - ⬭ 0

1. If three resistors total 15 ohms, and one is 9 ohms, what are the other two, if they are equal?

   ○ $15 + 9 = 2x$
   $x = 12$ ohms

   ○ $\frac{15}{9} = 2x$
   $x = 1\frac{2}{3}$ ohms

   ○ $\frac{2x}{15} = 9$
   $x = 6$ ohms

   ○ $15 = 9 + 2x$
   $x = 3$ ohms

2. If voltage equals current times resistance ($E = IR$), the current is equal to _____ .

   ○ $I = \frac{E}{R}$

   ○ $I = \frac{R}{E}$

   ○ $P = I^2R$

   ○ $R \times E = I$

3. What is the resistance when the voltage is 18 volts and the current is 12 amps?

   ○ 216 ohms

   ○ 30 ohms

   ○ $1\frac{1}{2}$ ohms

   ○ $\frac{2}{3}$ ohm

4. Which is Ohm's Law?

   ○ $E = IR$

   ○ $\frac{R}{I} = E$

   ○ $I = RE$

   ○ $R = IE$

5. Which is a correct solution to $[2(4x - 3) + 8 = 2(x + 10)]$?

   $x = 4$ $x = 2$

   $x = 2.5$ $x = 3$

6. If the power is 90 watts and the resistance is 15 ohms, what is the current?

   $\sqrt{6}$ amps   6 amps

   $\frac{1}{6}$ amp $\sqrt{75}$ amps

7. What size resistor uses 200 watts when carrying 8-amps of current?

   ○ 2-ohm

   ○ 4-watt

   ○ 3-ohm

   ○ 5-ohm

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1. When the frequency is 120 hertz and the inductive
    reactance is 3000 ohms, what is the inductance?
    $(L = \frac{X_L}{2 \pi f})$

    ○ 39.6 hy   ○ 3.98 hy
    ○ .396 hy   ○ 3960 hy

2. If the reactance were 2000 ohms and the inductance
    4.2 hy, what is the frequency required?
    ○ 76 hertz   ○ 80 hertz
    ○ 50 hertz   ○ 60 hertz

3. If the capacitance is 300 Mfd and the frequency is
    60 hertz, what is the capacitive reactance? $(X_c = \frac{1}{2 \pi fC})$
    ○ .112 ohms   ○ 6.28 ohms
    ○ 8.84 ohms   ○ 3.74 ohms

4. How much current is drawn when the power is 240
    watts and the resistance is 15 ohms?
    ○ 3 amps   ○ 4 amps
    ○ 5 amps   ○ 6 amps

5. How many poles are there in a 120-hertz alternator
    turning at 1800 RPM?
    ○ 4   ○ 2
    ○ 6   ○ 8

6. What is the wavelength of water waves traveling 75
    inches per second at a frequency of 15 cycles per second?
    ○ $\frac{1}{5}$ inch   ○ $2\frac{1}{2}$ inches
    ○ 5 inches   ○ 15 inches

7. What is the resistance when the power is 2640 watts
    and the voltage is 215 volts?
    ○ 10 ohms   ○ 15 ohms
    ○ 20 ohms   ○ 17 ohms

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1. Three resistors, R_1, R_2, and R_3, add to give a total resistance of 56 ohms. $R_1 = \frac{1}{3} R_2$ and $R_2 = \frac{3}{4} R_3$. What is the value of R_3 ?

   ○ R_3 = 28 ohms   ○ R_3 = 21 ohms
   ○ R_3 = 35 ohms   ○ R_3 = 25 ohms

2. What is the value of y in 4x + 3 = y when x = 2?

   ○ 3   ○ 6
   ○ 9   ○ 11

3. Which equations graph as straight lines?

   ○ parabolas   ○ hyperbolas
   ○ linear   ○ circular

4. How would 3x + 4y = 9 be solved for y ?

   ○ $y = \frac{-3x + 9}{4}$ ○ $4y = \frac{-3x + 9}{2}$

   ○ $\frac{y}{4} = \frac{x - 9}{3}$ ○ $y + 9 = \frac{x - 4}{3}$

5. What is a solution to x − y = 0 and x = 2y − 3 ?

   ○ x = 2, y = 1   ○ x = -1, y = 1
   ○ x = 3, y = 3   ○ x = 0, y = 0

6. A point on a graph is referred to by _____ .

   ○ coordinates   ○ referents
   ○ variables   ○ derivatives

7. How does the dependent variable y vary?

   ○ by moving to the right
   ○ with the value of the graph's origin
   ○ with the value of the independent variable x
   ○ with the total number of variables

8. What kind of slope points downward to the right?

   ○ positive   ○ negative
   ○ ambiguous   ○ hyperbolic

1. What is a solution to $x + 12y = 18$ and $x - 3y = 3$?
    ○ $x = 1$ ○ $y = 4$
    ○ $y = 9$ ○ $x = 6$

2. Two packages of resistors and capacitors are for sale.
    Thirty resistors and 20 capacitors sell for $4.50.
    Another package with 20 resistors and thirty capacitors
    sells for $3.50. Express this in equation form.
    ○ $20R + 30C = \$2.40$ ○ $20R - 30C = \$2.40$
       $30R + 20C = \$4.20$ $30C + 20R = \$4.20$

    ○ $30R + 30C = \$4.20$ ○ $30R + 20C = \$4.50$
       $20R + 20C = \$2.40$ $20R + 30C = \$3.50$

3. To solve for resistors, what is the first step?
    ○ multiply Equation I by 2 and Equation II by 3
    ○ multiply Equation I by 3 and Equation II by 2
    ○ multiply Equation I by 6 and Equation II by 5
    ○ divide Equations I and II by 9

4. What is the cost of a single resistor?
    ○ $.02   ○ $.04
    ○ $.05   ○ $.07

5. If we are given that $W = EI$, and $I = \frac{E}{R}$, how can W
    be expressed in terms of E and R?
    ○ $W = ER$ ○ $WE = R$
    ○ $W = \frac{R}{E}$ ○ $W = \frac{E^2}{R}$

6. What two numbers, x and y, add to give 84 and subtract
    to give 32?
    ○ $x = 58, y = 26$ ○ $x = 36, y = 48$
    ○ $x = 56, y = 28$ ○ $x = 50, y = 34$

1. What is the standard form of $6 - 4x = -3x^2$?
    ◯ $b + 3x^2 - 4x = 0$ ◯ $3x^2 - 4x + 6 = 0$
    ◯ $4x + 3x^2 - 6 = 0$ ◯ $3x^2 + 6 - 4x = 0$

2. What is the general form of a quadratic equation?
    ◯ $x^2 + x + 1 = 0$ ◯ $1 + x^2 + x = 0$
    ◯ $ax^2 + bx + c = 0$ ◯ $1 + ax - bx^2 = 0$

3. $x^2 - 25$ can be factored to $(x + 5)(x - 5)$. What is the value of x?
    ◯ 5.   ◯ ±5
    ◯ 25   ◯ ±2.5

4. Solve $x^2 - 5x - 36$ by factoring.
    ◯ x = +6   ◯ x = -4
       x = -6   x = +9

    ◯ x = -12   ◯ x = -18
       x = +3   x = +2

5. What is the quadratic formula?
    ◯ $\dfrac{c \pm \sqrt{a^2 - 4bc}}{2a}$

    ◯ $\dfrac{-a + \sqrt{c^2 - 2a}}{4b}$

    ◯ $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

    ◯ $ax^2 + bx + c = 3$

6. Solve $2x^2 - 5x - 3 = 0$ using the quadratic formula.
    ◯ x = 2, x = -4   ◯ x = 6, x = 3
    ◯ x = 3, x = -$\frac{1}{2}$ ◯ x = $\frac{3}{2}$, x = 0

7. Find a solution for $x^2 - 10x + 21 = 0$.
    ◯ +7, +3   ◯ -2, -4
    ◯ -7, +6   ◯ +6, +5

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1. What is the side opposite the 90° angle in a right
    triangle called?

    ⬭ arm   ⬭ base
    ⬭ hypotenuse   ⬭ leg

2. The three angles of a triangle always add up to _____ .

    ⬭ 180°   ⬭ 360°
    ⬭ 60°   ⬭ 90°

3. Which is the sine of an angle?

    ⬭ $\dfrac{\text{hypotenuse}}{\text{adjacent side}}$ ⬭ $\dfrac{\text{opposite side}}{\text{adjacent side}}$

    ⬭ $\dfrac{\text{opposite side}}{\text{hypotenuse}}$ ⬭ $\dfrac{\text{adjacent side}}{\text{hypotenuse}}$

4. Which function represents the adjacent side over the
    hypotenuse?

    ⬭ tangent   ⬭ cotangent
    ⬭ cosecant   ⬭ cosine

5. What function solves for c ?
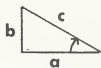
    ⬭ tangent   ⬭ sine
    ⬭ secant   ⬭ cosine

6. What are the sine and cosine of 45° ?

    ⬭ sin = 1   ⬭ sin = .5
      cos = 0   cos = .866

    ⬭ sin = .707   ⬭ sin = .866
      cos = .707   cos = 1

7. How many degrees are in an acute angle?

    ⬭ 45°   ⬭ 90°
    ⬭ 240°   ⬭ always less than 90°

1. Which quantity specifies amount and direction?

    ◯ scalar   ◯ linear

    ◯ modal   ◯ vector

2. What is another name for the polar-coordinate vector system?

    ◯ r-theta system ◯ half-angle circular system

    ◯ $\pi^2 r$ system   ◯ d-triangular system

3. How many basic rectangular components are there to a vector?

    ◯ 1   ◯ 3

    ◯ 2   ◯ 4

4. What function represents "the side opposite the angle over the hypotenuse?"

    ◯ tangent   ◯ sine

    ◯ cosecant   ◯ cosine

5. What is the magnitude of a vector with an x-component of 6 and a y-component of 8?

    ◯ 10   ◯ 7

    ◯ 4.5   ◯ 2

6. Which function represents "the side adjacent to the angle over the hypotenuse?"

    ◯ cotangent   ◯ sine

    ◯ cosine   ◯ cosecant

7. What function would be used to find the x-component of a vector when the angle and the y-component are given?

    ◯ cosine   ◯ cogent

    ◯ tangent   ◯ sine

Arithmetic Operations

1. Electronics is a "quantitative" technology, that is, we need to know "how far," "how high" and "how much," in nearly every situation. The relationships of the quantities are often complex, and not easy to express in ordinary ways. For these two reasons, you will find it necessary to learn and practice the procedures of mathematics, including arithmetic and algebra.

2. This series of 16 audiovisual teaching machine programs is designed to help you learn and review those principles of mathematics which are most useful to electronics technicians and engineers. Occasionally, you will apply math to problems which are typical of electronics. For example, you may have two loudspeakers using 8 watts each and two loudspeakers drawing 5 watts each in your four-speaker set-up. What is the total power required? (18 watts) (21 watts) (26 watts)

3. Right. Of course you may be able to do this sort of thing in your head, or on scratch paper, but a little review and practice won't hurt. Most of these programs proceed rather rapidly, and you may wish to repeat them two or three times.

4. Even though you understand some arithmetic, or other math procedures, however, you'll need a real facility in using them quickly and accurately, so a careful, thorough review is worthwhile. For example, what's the sum of plus 5 and minus 3? (2) (5) (8)

5. Yes. You'll learn about positive and negative, or "signed" numbers, later. You won't have much time in these programs, however, for extended practice, for it is assumed that you have previously studied arithmetic and need only a short review to refresh your memory. If you need more, there are more than 100 available programs on various mathematical operations.

6. Later programs in this series will teach you to use the slide rule and electronic calculator, and how to do rapid arithmetic calculations in your head. All of these ways to add, subtract, multiply, divide, square numbers and take square roots of numbers are useful, and you should know about them. What's the approximate square root of 50? (5.2) (7.1) (25)

7. Correct. A little practice with the multiplication table will help improve your accuracy and speed. Facility with arithmetic will give you the capacity and the confidence needed to approach all kinds of logical problems in electronics.

8. In the future, the availability of small, portable calculators will make it less necessary to practice mental or pencil-and-paper addition of long columns of figures. You should do some of this occasionally, however, to be sure to keep in practice. Write down three or four numbers

each of several digits and add, then check by using a calculator or adding machine. Try to answer this quickly. What's the total of 18 plus 25? (33) (34) (43)

9. Right. Remember, our system of numbers is composed of the ten digits, and in numbers greater than 9, two or more digits are required, and each digit to the left of the right-hand digit has a place value ten times as great as the same digit in the right-hand place. There is the units place, the tens place, the hundreds place, and so on.

10. When the sum of a column of digits is more than nine, you must write down only the right-hand units-digit in the total, and carry over, to the next left column, any higher digits.

11. Also, don't forget, you must be sure that you are adding volts to volts, amps to amps or ohms to ohms. If I have two apples, you have 3 oranges, what do we have? (5 apples) (5 oranges) (2 apples and 3 oranges)

12. For just one practice exercise, what's the total load on an electrical circuit which has a TV set using 228 watts, and two lamps, one with a 75-watt, and one with a 60-watt bulb? (363 watts) (397 watts) (433 watts)

13. Yes. Subtraction means finding the difference between two numbers. Subtraction is the inverse operation of addition, so when you can add quickly and easily, you're on your way to subtracting easily. But while you can add several numbers more or less at the same time, subtracting always involves only two numbers.

14. If you wish to be formal, which those who work in electronics seldom are, you might say that an addend plus an addend make a sum, or total, and a minuend less a smaller number called a subtrahend gives a difference. Sometimes the difference is called the remainder. Let's not use any of these words except sum and difference, but for convenience in subtracting on paper, we'll always write the larger number above the smaller one, whatever we call them.

15. At this point you should be reminded that negative numbers are used in some arithmetic and algebra problems. While most arithmetic numbers are considered positive numbers greater than zero, some numerals are used to represent quantities less than zero, or at least on the opposite side of some arbitrary reference point. How would you write 40 degrees below zero? (40°) ($\frac{1}{40^{\circ}}$) (-40°)

16. Right. In most cases, you may omit the sign on numbers, primarily because they may be assumed positive, or it's irrelevant. But when it is relevant, it's quite important, for a substantial error may be introduced if you add a number which should be subtracted.

17. When you multiply or divide two numbers and both are either positive or negative, the result is always positive. But when one is positive and the other negative, the result is always negative.

18. But when you add two or more numbers, or subtract one from another, don't forget that the sum or difference resulting may be either positive or negative, depending upon the size and sign of the numbers which are involved.

19. The need for special care in watching signs results from our usual practice of ignoring the assumed positive sign for most numbers. Also, since we use the plus and minus signs both to indicate the positive or negative direction of numbers, and to specify the arithmetic operations of adding and subtracting, we can easily get confused.

20. For example, we may need to subtract a negative number from a positive number. Arithmetically, we find this is just addition, but you can see that we must use some care if a series of similar operations are done, and the values change from positive to negative, or the reverse. What is the result of subtracting a negative 3 from a positive 7? (+4) (+10) (– 10)

21. Yes. What is the product of minus 6 times minus 5? (– 11) (– 30) (+30)

22. Right. What is the product of plus 4 times minus 3? (+12) (– 12)

23. What is the sum of a positive 17 added to a negative 12? (+5) (– 5) (– 29)

24. Yes. What is the difference when – 25 has – 12 subtracted from it? (+13) (– 13) (– 37)

25. In other programs, you will learn how to use a slide rule, an electronic calculator, and quick mental arithmetic. You will always need to perform quickly, certain arithemtic operations with simple numbers, either in your head or on paper, so if you're a little rusty, take some time to practice. For example, try these.

26. What's the product of 7 times 9? (63) (72) (79)

27. Quickly, what's 8 times 7? (49) (56) (78)

28. What's 9 times 12? (98) (108) (118)

29. You'll probably find that there are several products in the multiplication table up to 12 times 12, which you may take a few seconds to remember. If so, a little refresher drill may save you time and perhaps embarrassment in your work in electronics.

30. Occasionally you may need to do some long-form multiplication and division. For a brief review, in multiplying, don't forget the place-value of integers. Take the product of each digit in the multiplier times each digit in the top number, or multiplicand, adding the tens digit each time to the product of the next left digit, and so on through the first product. In this case, we are multiplying 6 times 234.

31. Then take the product of the next left multiplier digit and repeat the process, offsetting it to the left one place. Why do we do this? (The next left digit has ten times the value.) (It's just arbitrary. The teacher said so.)

32. Yes. Then, of course, the offset, individual line products are added to obtain the total product, making sure to carry any tens digits. No doubt you've done this most of your life, or you intend to keep a calculator, but you need to polish up your basic skill in doing it.

33. Remember, long-form division offsets its place-value-subtraction steps to the right, which is the opposite of the product-adding procedure in multiplication. In the problem 13, 104 divided by 56, what will be the first digit in the quotient? (2) (3) (4)

34. What is the next digit we should try in this quotient? (2) (3) (4)

35. Yes, here it is. What's the complete quotient? (123) (234) (456)

36. Let's say you need a rug and it costs a dollar a square foot. Can you afford it? Let's see. What's the product of 8 feet times 12 feet? (96 square inches) (108 square feet) (96 square feet)

37. Right. You decide to lower your room temperature eight degrees to save the energy of the heating fuel. What's 76 degrees minus an 8-degree change? (68 degrees) (72 degrees) (84 degrees)

38. Let's say you brought four apples, 6 oranges and 5 bananas to the picnic. Your friend brought you 3 apples and ate one of your bananas. Yow much fruit do you have? (18 pieces of fruit) (7 apples + 6 oranges + 6 bananas) (7 apples + 6 oranges + 4 bananas)

39. Right. You have 4 xerox copies and 5 yellow carbons, and your boss takes 2 xerox copies and gives you two yellow copies. What do you have?

$$\begin{array}{r} 4x + 5y \\ +(-\ 2x + 2y) \\ \hline 2x + 7y \end{array} \qquad \begin{array}{r} 4x + 5y \\ -\ 2x - 2y \\ \hline 2x + 3y \end{array}$$

40. You drove 100 miles and used 8 gallons of gas. What was your rate of using gas? (18 miles/hr.) (12½ miles/gallon) (12½ m/h)

41. Yes. This quick introduction to the series of electronics math programs is simple, but before you are through, you'll learn some quite useful skills.

1. Electronics engineers and technicians have used the slide rule for many years to save time, avoid errors and achieve adequate accuracy for electronic problems.

2. You can learn very quickly to use the slide rule for performing multiplication, division, unit conversion, and to look up various functions. But there are so many kinds of slide rules, and so many ways they can be used, that it would take a long time to learn about them all.

3. In this program you'll learn some of the most common ways to use the slide rule, and probably the more complex procedures will in the future be performed by calculator or computer.

4. The slide rule is a mechanical means of presenting a table, and of adding and converting. You could place two yardsticks side by side, placing the beginning of one on, say, the 3-inch point of the second one. Then all of the points on the second yardstick would be 3 units greater than the ones just above them on the first yardstick. This would be adding, but it's hardly worth the trouble.

5. If you placed the left index, or the left end of a special logarithmic scale on a point, say, 3, on another such scale, you'd find that the points on the second scale aren't just 3 units greater. They're 3 times as great! What operation is performed this way? (addition) (multiplication) (division)

6. Yes. A logarithmic scale is one where proportional increases always occupy the same space. The same distance, for example, is devoted to the space between 2 and 4, or 3 and 6, as between 1 and 2. Also, the same distance occurs between 2 and 3, or 4 and 6, as between 1 and 1½.

7. This is a special electronic slide rule with several special scales in addition to the so-called "C" and "D" scales which are used for most multiplying and dividing. At the bottom is the logarithm, obtained by reference to the C or D scales. The Cf and Df scales are "split scales," beginning and ending at the value "pi," for quick multiplication by that number.

8. Although you'll learn in this program how to use the common scales, for using the slide rule for special purposes you may wish to check with your instructor, or on the job, with your senior associates. As with other instruments, if all else fails, what do you do? (read the instructions) (give up)

9. Yes. For use in electronics, the accuracy of a slide rule is usually far in excess of that really needed for nearly every application. A little care in using a 10-inch slide rule will provide results good to one part per 1000. What percentage error is this? (.001%) (.1%) (100%)

10. Correct. It's rare that you'll be working with resistors, for example, with precision greater than 1%; and plus or minus 10% resistors are far more common. Ordinary meters are accurate only to 1 or 2%. Thus the 10-inch slide rule is amply accurate, and a 6-inch rule will usually provide useful answers.

11. It is important not to imply to others, by the use of several extra digits, that the data you have is more accurate than it actually is. Every non-zero digit in a number is a "significant digit." 3.1416 has five significant digits. How many significant digits does this number (.00257) have? (3) (4) (5)

12. Yes. How many significant digits does this number have? ($6,500,000,000) [2] [7] [9]

13. You should learn to "round off" numbers which appear to represent more accuracy than you believe they have. In the program on "how to do 2% arithmetic in your head," you'll get some practice in rounding off numbers.

14. Let's say you need to find the voltage across a 47-ohm resistor with 1.25 amperes flowing through it. This means you must multiply 47 by 1.25. First set the left-hand index, that is, the "1" mark of the C-scale, on the 1.25 point of the D-scale by sliding the slide to the right. Has this been done on the slide rule shown? (Yes) (No)

15. Yes, although there is no number, just a mark, at 125, it's obviously the mark which is half-way between one-two and one-three. Now look at the point for four-seven on the C-scale. The index line on the glass of the cursor is set on it. What is the value just below it, on the D-scale? (47.8 volts) (53.8 volts) (58.75 volts)

16. Right. Before we change the position of the slide, what is the product of 1.25 times 4? (4.8) (5) (6.4)

17. When the left-hand index of the slide is set on a multiplier such as 7, 8, or 9, there is not much space where the C-scale overlaps the D-scale to find the product, so you may find it necessary instead to place the right-hand C-scale index, or scale end, on the multiplier on the D-scale, like this. With the multiplier set on 7.5, what is 7.5 times 8? (56) (60) (64)

18. Here the right index of the C-scale is set on 95. What is 95 times 82? (7790) (7562) (8160)

19. Yes. Of course you may not always be able to read to the accuracy of three significant figures, for when the left digit is 7, 8, or 9, this approaches one part in 1000. What is the product of 2.25 times 128? (275) (282) (288)

20. You may wonder if there isn't some way that the nuisance of checking to see which C-scale index to use can't be eliminated. There is. One way is to use the A and B scales on the other side. Another is to use a circular slide rule, which we'll describe later. Most users prefer the linear scale, even with the occasional need to shift ends, over the circular scale. And if you use the A and B scales, your accuracy and ease of reading the scale is reduced.

21. You have just done some multiplication in which there was no need to worry about decimal places. But unfortunately, working with electronics involves an extremely wide range of values, and it's all too easy to make a mistake in a decimal place. Frequencies, resistance, capacities and impedances may vary a million to one.

22. A capacitor may be specified in picofarads, or a millionth of a millionth of a farad. And a megohm is a million ohms; a megahertz is a million cycles per second. How many ohms are there in a 2.7 megohm resistor? (2700 ohms) (270,000 ohms) (2,700,000 ohms)

23. Right. The important rule to remember is that when expressing a given decimal fraction as a number between 1 and 10, times some power of 10, move the decimal to the right, and count the number of places from the original point. This number of places is the negative power of 10.

24. For example, how could you express the value of a capacitor with .00012 microfarads? $(1.2 \times 10^{-4}$ Mfd) $(1.2 \times 10^{-5}$ Mfd) $(1.2 \times 10^{-6}$ Mfd)

25. How would you write the value of 810,000 ohms? $(8.1 \times 10^5$ ohms) $(8.1 \times 10^6$ ohms) $(8.1 \times 10^7$ ohms)

26. You may need a lot of practice to become adept at handling powers of 10. If so, you may wish to study the program on this subject. An important step in using your slide rule, or in any kind of calculation, except perhaps in an electronic calculator, is to look at the result and decide if it makes sense. If it seems too large or small, you should recheck your decimal place.

27. Division is the inverse of multiplication, of course, and it's easier than multiplication on a slide rule, since you don't have to decide which C-scale index you need to use. It's decided for you. First set the cursor index on the dividend, which is the number being divided, located on the D-scale. Then set the divisor on the C-scale at the cursor index, then read off the answer on the D-scale, under the index, or end, of the C-scale which touches the D-scale.

28. Here we are dividing 27 by 15. Which scale has the 27? (C-scale) (Cf-scale) (D-scale)

29. Right. At one end of the C-scale, its index points to the quotient. What is it? (1.75) (1.80) (1.85)

30. Yes. We wish to divide 275 by 25. How do we arrange the cursor? (cursor on: 25 on C-scale, 275 on D-scale) (cursor on: 275 on C-scale, index of D-scale) (cursor on: index of C-scale, 275 on D-scale)

31. Yes. Don't forget; the number being divided, or the dividend, and the quotient, or result, both appear on the D-scale, the dividend under the divisor and marked by the cursor, and the quotient under the C-scale index.

32. You can't develop speed and accuracy in using the slide rule without an extended period of practice, of course. If you wish to gain such proficiency, write down a series of problems and perform them, checking each result. A calculator will help you check results. In fact, you may decide that a small calculator is better than a slide rule. If so, the next program in this series will be of interest to you.

33. You will notice that the A and B scales have two series of numbers between 1 and 10, compared to the C- and D-scales which have one series each. You can multiply and divide using these two upper scales, but they are principally useful for squaring and taking square roots. For example, set the cursor on the middle "1" mark on the B-scale. This may be considered "10" for square root and squaring purposes. What is the square root of 10 shown on the D-scale? (3.14) (3.15) (3.16)

34. Right. What's the square root of 20? (4.42) (4.47) (4.52)

35. Yes. The "Ci" scale is the inverse of the C-scale. Inverse numbers are often useful, but the obvious result of using the Ci scale with, say, the D-scale, is that you invert the operation. Using the multiplication procedure you learned for the C and D scales, with the Ci and D-scales, will result in division. Using the C-inverse-to-D division procedure will perform multiplication.

36. At the top of the C-D side of most slide rules is a logarithm, or "L" scale. Since the C- and D-scales are logarithmically scaled, the log scale is linear! On a slide rule with ten-inch scales, the L-scale is just a finely divided 10-inch ruler!

37. Logarithms can be used to work with fractional exponents and the like, but your use for them may not be very great for some time. These are base-ten logarithms, and their use is explained later.

38. On many slide rules, including special electronics slide rules, there are scales for trigonometric functions, such as sine and tangent. These functions are explained in another of these electronics math programs, and their use is discussed from time to time in the electronics theory programs. These scales are the "S" scale, and "T" scale and sometimes a folded "ST" scale for small angles.

39. On some slide rules designed for electronics, there are special scales for impedance and other values. Their use will be mentioned as they are studied. Meanwhile, the ability to multiply and divide with a slide rule will be useful. You may find it particularly handy to perform a series of such operations, for example, $\frac{2\pi R^2}{36}$, where R is, say, 7.25. Perform each multiplication first. If you put the left C-scale index on π on the D-scale, then moved the cursor to 2 on the C-scale, what would it read on the D-scale? (6.14) (6.16) (6.28)

MATHEMATICS FOR ELECTRONICS   Me 3
Using an Electronic Calculator

1. The most important development which has occurred in the use of arithmetic in the past century has been the development of small, handy, low-cost portable calculators. As an electronics expert, you will have some part of this important and exciting field, and as a technologist, you will need to use such calculators frequently to solve mathematical problems in electronics.

2. In this program, you will learn how to use an electronic calculator for addition, subtraction, multiplication, division and chain calculations. You will also learn a little about such special tricks as obtaining square roots and percents without a special program or key for them.

3. The simplest calculators have keys for the 10 digits of 0 through 9, decimal point and clear keys, and the four operation keys, +, -, x for times and the "divide" sign. This totals 16 keys. A desirable additional key is a "CE" or "clear entry" key so that you don't have to clear the entire register if you list a wrong key while entering a new number after one or more numbers have already been entered.

4. You have a minimum calculator. It has 16 keys, including the 10 digits, the four operations keys and a clear key. What other key does it have? (percent) (constant) (decimal)

5. Right. The electronic calculator is easy to use and provides accurate results if properly operated. Its results are accurate to the nearest visible digit. This may lead to the impression that the results of electronic calculations are more precise than they actually are, since most observed data and component specifications are not as accurate as the calculations. For this reason, it's important to round off most results from the calculator to two or three significant figures.

6. Most small electronic calculators have an on-off slide or toggle switch. They may have 6 to 12 digits in their display, plus intervening decimal points, and probably some means of indicating when the display is overloaded. Actually, if you have a convenient way of shifting the display to the right or left, six figures would usually be enough for electronics, if not for financial purposes. This is true because most electronics calculations only require moderate precision. Usually, how accurate are they? (not more than three significant figures) (6 significant figures) (one part per million)

7. Yes. Let's say that you have two resistors, a 270-ohm resistor and a 680-ohm resistor connected in series. This provides a total effective resistance which is the sum of these two resistors. You must add the two numbers. What do you do first? (enter 270) (clear the display) (switch on the calculator)

8. Right, first switch on the calculator, then clear the display, and press the 2, the 7, and the zero buttons. Then press the "plus" button, which may say "plus and equal." Then press the 6, the 8, and the zero buttons. What button do you think you should press next? ($\overset{+}{=}$) (x) ($\div$)

9. Yes. This would give you the answer, 950, perhaps with a decimal point and two more zeros. This may occur even if you didn't enter the decimal, if the calculator has a fixed setting. Assuming the unit is on and clear, adding two or more positive numbers is done by entering the number and pressing the "plus-equals" button after each number entry.

10. Adding negative numbers is easy, too. You just clear, enter the number and press the "minus" key. Obviously, if the first number you enter is a negative number, the entry total will be negative, and in most calculators this will be denoted by a minus sign to the left of the display register.

11. Let's say you notice a total current flow of 425 milliamperes, and find that 72 milliamps of it is flowing in one circuit parallel to another. What is the current in the second circuit? Clear, enter 4, 2, 5, press plus-equal, then enter 7, 2, then what key? ($\overset{-}{=}$) (x) ($\overset{+}{=}$)

12. Yes. The result 353, or 353.00, will be shown immediately. The difference is a positive number, so there won't be any sign in front of the register, or at least no minus sign.

13. You need to subtract 32 from 96. You enter 96, press +, enter 32, press –. What did you forget to do? (enter 32 first) (press x) (clear)

14. Right. Chain addition or subtraction is simple. Just clear first, then enter each number, and follow with its sign. The subtotal will be shown after pressing each sign key. Clear, enter the number, press "plus" for adding a positive number, or "minus," if you are either adding a negative number or subtracting a positive number.

15. Multiplication is easy, too, but you must be careful to do things in the correct order. Clear, enter the first number, which we'll call the multiplicand, press the "x-for-multiply" button, then enter the multiplier number, then press the plus-equals button. You can read the product directly.

16. Let's try one, 12 times 15. First we clear and enter "12," then press "x-for-multiply," then, "15," then "plus-equals." What's the result? (152) (180) (258)

17. Yes. This lamp's current is 7.5 amps, and the car voltage is 12 volts. What's the power in watts? Clear and enter "seven." Then what? (5) (x) (.)

18. Right. Then the 5, for 7.5 amps, then "x-for-multiply," then 1 and 2 for 12 volts, then what? ($\overset{+}{=}$) (–) (x)

19. Yes. Since some more complex calculators like the HP-35 or HP-45 have keys which operate on unknowns, and the unknown is designated "x," we said "x-for-multiply" since the x-shaped multiplication sign is similar to the x for unknown. If you look carefully, you can generally see a difference in character shape, however.

20. Let's assume you wish to find the surface area of a 7-inch phonograph record. Clear, enter 2, press "x-for-multiply," enter 3 decimal 14 for "pi," press "x-for-multiply," enter 3 decimal 5 for the radius of the record, press "x-for-multiply," enter 3 decimal 5 for the radius squared. Then what? ($\pm$) (C) (0)

21. Yes. On some calculators you'd get the result, 76.93, if you pressed any of the operations keys except minus. That is, you may get the result displayed by pressing "x-for-multiply" or the divide button, as well as "$\pm$" since this would bring up an interim-display. On the other hand, such a divide or multiply operation would then prohibit further chain operations with the display, except, respectively, dividing or multiplying.

22. Let's say you wish to know the current through a 120-ohm resistor when it's connected across 45 volts. You divide 45 volts by 120 ohms. Clear, enter 45, press the "divide" button, enter 120, press "$\pm$." What do you get? (3.75) (0.37 or 0.375) (2.66)

23. Yes. In this case on some calculators, you'd get only two significant digits and read "point 37 amps," or "370 milliamps," when the precise answer is 375 milliamps. If three significant digits are really necessary, you could set in an extra zero or two in the dividend, or drop a decimal or two in the divisor, but remembering to put them back in, in the quotient, as with a slide rule. If you have a floating, or adjustable decimal calculator, of course, you don't have to do this.

24. The power in a nonreactive load can be calculated, even if you don't know the current, if you do know the voltage and resistance. It's equal to voltage, squared, divided by resistance in ohms. Let's say a resistor of 8 ohms is connected to a 12-volt automobile supply. What do you do to find power in watts? (multiply volts times ohms) (divide volts by ohms) (multiply volts times volts; divide by ohms)

25. Right, 12 times 12 divided by 8. Clear, enter 12, press x, enter 12, press "divide," enter 8. Then what? (press $\pm$, get 18.00) (press x, get 15.00)

26. Yes. But what if we already knew the power and the resistance, and wanted to find the voltage? Say we had a one-ohm resistor which was rated at 475 watts; what is its rated voltage? Well, we would multiply 475 by one, and take the square root of it. How do we use the calculator to get the square root of 475?

27. Here's how. Point off two places on 475. That makes 4 point 75. What's the square root of 4? (2) (3) (4)

28. Of course, two. Then the square root of 475 is something more than 20. Here's how to find a better value for it: divide 475 by 20, and average the result with 20. Clear, enter 475, press "divide," enter 20, press "plus-equal," get what? (23.75) (24.75) (27.45)

29. Yes. The square root of 475 is approximately the average of 20 and 23.75. Now before

clearing, enter 20, press "plus-equal" which gives 43.75. Then press "divide by," enter 2, then press "plus-equal." What will you get as the approximate square root of 475? (20) (21.87 or 21.875) (23.75)

30. Correct. You can get even closer by dividing 475 by 21.87, to get 21.71, then add 21.87 again, divide by two and get 21.79. It would appear that 21.8 is a very close approximation of the square root, and is essentially achieved by a single averaging operation on a rough estimate of the square root with the resulting quotient.

31. Let's have a little review drill. How do you add 243 and 721? (clear, enter 243, $\pm$, enter 721, $\pm$, read 964) (clear, $\pm$, enter 243, $\pm$, enter 721, $\pm$, read 964)

32. Yes. How would you subtract 243 from 721? (clear, enter 243, $\pm$, enter 721, – , read 478) (clear, enter 721, $\pm$, enter 243, – , read 478)

33. Right. What's the voltage across a 150-ohm resistor passing 1.25 amps of current? (C, 150, x, 1.25, $\pm$, read 187.5) (C, 150, $\pm$, 1.25, $\pm$, read 151.25)

34. Yes. What's the current through a 5-ohm resistor across a 12-volt supply? (C, 12, $\div$, 5, $\pm$, read 2.4) (C, 5, $\div$12, $\pm$, read 2.4)

35. What's the current through a 15-ohm resistor which is using 32 watts? First, we are told that watts equals amps, squared, times ohms resistance. By examination, we say that amps, squared equals watts divided by ohms. So we'll do this division first. How do we divide 32 watts by 15 ohms? (C, 32, $\div$, 15, $\pm$, read 2.13) (either way) (C, 32, $\div$, 15, $\div$, read 2.13)

36. Yes, on most calculators you could get the same readout either way, but since you are going to go ahead and get the square root of the first quotient, which represents amps, squared, you might as well just press the division button and divide by an estimated square root of 2.13.

37. What's your estimate of the square root of 2.13? Somewhere between one and two, of course. Trying 1.5 would be fine. You should probably memorize the square root of 2, which is 1.414. So you could divide by 1.4. Let's try it. What do you do? (C, 2.13, $\div$, 1.4, $\pm$, read 1.52) (have 2.13, $\div$, 1.4, $\pm$, read 1.52)

38. Right. You already have the dividend in the calculator. Now, to average the resulting quotient, you leave it in the register, add back your divisor 1.4, and get 2.92. Then what? (have 2.92, $\div$, 2, read 1.46) (have 2.92, $\pm$, 2, read 4.92)

39. Yes, the square root of 2.13 is 1.46. You would have obtained the same answer in the same single cycle if you had used 1.5 as the trial divisor. You would achieve greater accuracy by pointing off 2 or 4 more decimal places, but in electronics it would be misleading, since your data is not that accurate.

"2 Percent" Arithmetic

1. In previous programs you have reviewed the fundamental arithmetic operations and seen how a calculator and a slide rule can help you make speedier calculations. This is a program which will help you do useful arithmetic operations rapidly in your head.

2. Electronics engineers and technicians aren't like bookkeepers. The numbers they deal with are seldom precise. The components they select from a bin usually differ from those in the next bin by at least 20%. For example, if you were out of 1000-ohm resistors, you could probably use a 1200-ohm resistor in the circuit, and chances are that it would make little difference.

3. Since this is true in most cases, why worry excessively if your electronic calculations result in 1010 ohms instead of 1000 ohms? It is very likely that the data you started with was accurate only to about 1%, and possibly even less. If you picked up a 1500-ohm resistor which actually measured 1550 ohms, about what error would you make in using it? (1%) (2%) (3%)

4. Right. However, there is a way or method of getting accurate enough answers. You just don't guess at the solutions to electronic arithmetic problems, because you might make errors of 25%, 50%, or even 100%. And if you made a decimal point error, your result could be off 1000%.

5. Occasionally, there is a kind of arithmetic problem called an "incremental" problem where you need to find the direction and amount of a small change in the answer, even though the exact value is not precisely required. In this case, you will need to use precise arithmetic procedures: adding a column of figures, carrying places, and checking your results, or subtracting two figures, borrowing from the next place, if necessary, and checking your results.

6. For multiplying, you know to apply the multiplication table one digit at a time from your multiplier, offset each line product, and add the results. For dividing you simply invert this process. There are lots of drills and practice exercises you can do. For example, what is the product of 27 times 33? (567) (891) (961)

7. In electronics, though, you should be able to do a lot of arithmetic in your head, especially if you realize that most of the data you start with is not really precise or if you have numbers which are given with useless precision. In these cases, you can round them off from 1% up to 10%. Make sure, though, that you don't introduce more than a 5% error, or an average error of 5%.

8. You know that 1% of 100 is 1. If you rounded 100.5 down to 100 or up to 101, you'd be making an error of $\frac{1}{2}$ of 1%. If you rounded 10.5 down to 10, you'd introduce a 5% error.

Rounding off 80.5 to 80 would produce an error of about .6 (six-tenths) of 1%. What kind of error would rounding off 50.5 to 50 make? (10%) (1%) (.1%)

9. We will call this method "2% arithmetic." In using it, you must first round off all numbers to two significant digits. Values like 12.36, 24.8, 39.78, 52.9, 93.175, 178.6, and 285.32 would then become numbers like 12, 25, 40, 53, 93, 180, 290 respectively. This is what we do in the color code rating of resistors. There, two color bands are used for the first two significant numbers and the third color band represents their place value.

10. What do .139, 629.6, 28.1, and 27,822 round off to? (.136, 630, 29, and 27,800) (.14, 630, 28, and 28,000)

11. Of course. By rounding off numbers to two significant digits, you are making a maximum error on one end of each series of 5% and a maximum error at the other end of ½ of one percent. The average error in this procedure is about 2½%. That's why we call it "2% arithmetic."

12. It is obvious how rounding off helps in addition and subtraction. You don't have to do so many columns of adding and carrying, or subtracting and borrowing. About what percent of error is involved in rounding off and then adding 5541 and 8322? (40%) (4%) (less than ½ of 1%)

13. Yes. The real saving is obviously in multiplication and division. These are the operations you will probably be doing the most. We assume you know the multiplication table to 12 x 12 = 144. If not, you should review it.

14. For example, what is the distance, expressed in feet, to the moon? Assume the distance in miles to be 235,000 miles with 5,280 feet per mile. Rounding off gives you 5,300 feet times 235,000 miles or 53 times 23, with 2 plus 4, or six zeros added. This gives a result of 1,219,000,000 feet. What would the result be without rounding off and what percentage error is introduced by rounding off? (1,240,800,000 feet; 1¼%) (1,296,000,054; 3%)

15. Right. The moon doesn't stay exactly 235,000 miles from us, though; that's just the average distance. If you want to go there, you'll need a computer for accurate navigation. But if you want to calculate the power or sensitivity of a radio to send or receive messages, "2% arithmetic" is adequate.

16. You know a resistor is rated to handle a power load of a half-watt. Its resistance is 1,000 ohms and the voltage is 15.25 volts across it. "Two-percent arithmetic" will help you find if the resistor is overloaded.

17. The formula is watts of power equals voltage, squared, divided by the resistance. Here that would mean that the power in the resistor is equal to 15.25 times 15.25 divided by 1,000. Round off the voltage to 15 and square it to give 225. You should memorize this value and some others you'll learn.

18. Dividing 225 by 1000 gives .255 watt, or safely less than the resistor's half-watt rating. If you had rounded off 15.25 to 20, would your decision have been the same? (Yes) (No)

19. If you know that voltage equals resistance times current, and that your current is 18.25 amps and the resistance is 12.75 ohms, what voltage does rounding off these values to 18 amps and 13 ohms give you? What percent error is introduced? (234 volts, 1%) (239.6 volts, 3%)

20. Yes. Let's try another example. You need a resistor which can carry a current of 0.875 ampere with a voltage of 4.15 volts. You want to know whether to use a resistor rated with a power capacity of a quarter-watt, a half-watt, one watt, 2½ watts, 5 watts, or 10 watts. Watts of power equals amperes times volts. What power rating of resistor should you use?

21. To figure this, we multiply 0.875 times 4.15. Try rounding 0.875 to .88 and 4.15 to 4.1 and multiply to get what amount of power? Which resistor should you use? (3.6 watts, 5 watts) (3.6 watts, 10 watts) (3.608 watts, 5 watts)

22. Incidentally, it is usually hardly worth the usual roundoff procedure for finding the decimal place, since in checking your answer, you can think of the problem in this way: ".88 is nearly 1, and 4.1 is nearly 4, so the answer is nearly 1 times 4, or 4." This inspection method roughly checks the numerical result, and provides the basis for inserting the decimal.

23. You might wonder why in electronics we don't round off to the nearest single significant digit, since in nearly every case this would result in the same final decision and action by an electronics technician. For example, if you had rounded the .875 amp current to .9 amp and the voltage from 4.15 to 4 volts, their product would still be 3.6 watts and you would have still needed the 5-watt resistor.

24. In this example, like most such cases, it made no difference how you rounded off to the nearest significant digit. But there are obvious cases where the direction and amount of rounding must be considered carefully. For example, where you have, say, 1.4999 amps and 1.4999 volts, rounding them both down to 1 amp and 1 volt would give a product of 1 watt. Comparing your answer with the precise calculation of 2.247 watts would show that you were throwing away too much in each round off.

25. Which would be the most proper way of rounding 1.4999 amps and 1.4999 volts to one significant digit to give a reasonably accurate answer? (Round both values down to 1.) (Round one value down to 1 and the other up to 2.) (Round both values up to 2.)

26. Right. In the case of rounding 9.5 amps times 9.5 volts to 9 times 10, you'd arrive at 90 watts rather than 90.25 watts. Even rounding them both down to 9, or both up to 10, would give you 81 or 100 watts. In any case, you'd pick a 100-watt rated resistor, or maybe even a larger rating, if you are inclined to have a substantial overdesign margin.

27. You can see that for most purposes in electronics, you can sensibly round off multipliers to one or two digits without any effect on the decisions you'd make in component selection. When

the left-hand, or most significant digit, is one or two, and rounding off would make considerable difference because the value is near 1.5 or 2.5, times 10 to any power, then you should probably use two significant digits: otherwise, one significant digit is adequate for most electronics purposes.

28. Let's try some representative examples. The current through a silver-banded 2200 ohm resistor is 4.95 milliamps. What's the voltage?

29. A silver-band resistor usually varies within 10% of its indicated value, although it is probably within 3 or 4% of the marked value. Then why not round off the 4.95 milliamps to 5 milliamps? We could even round the resistance to 2,000 ohms and not change the value more than 10%.

30. In this case, the decimal place steps can be made easier by remembering that while volts equals amps times ohms, volts also equal milliamps, or thousandths of an amp, times kilohms, or thousands of ohms. This means that you can multiply 2.2 kilohms, or 2.2 K-ohms times 5 milliamps and get 11 volts. You would get 10 volts if you also rounded the resistance to simply 2 K ohms.

31. What was the percent error caused by each rounding off; was the average percent error within the 10% tolerance for the resistor? (1%; 8%, yes) (5%; 20%, no)

32. Correct. Suppose we want to know the resistance in a circuit where the current is measured at 6.85 milliamps and the voltage across it is 32.2 volts. Which would be the best choice for rounding off both values to one significant digit? (6.85 to 7, 32.2 to 30) (6.85 to 6.9, 32.2 to 33) (6.85 to 6, 32.2 to 40)

33. Yes. We get resistance in K-ohms by dividing voltage by milliamps, or 32.2 over 6.85. A quick estimate would give us rounded values of 30 over 6 or 5 K-ohms. More accurately, it is 32 over 7, or $4\frac{4}{7}$. Expressed in decimals, since we know that $\frac{4}{7}$ is a little more than ½, we might say the resistor is 4.6 ohms.

34. Going back to our calculator, we find 4.7007 K-ohms, which tells us that somehow we've run across a standard-value 4.7 K resistor with a remarkably precise value, or at least there was a re-markable coincidence in the meter readings. Since the standard resistor values below and above are 3.3 K and 6.8 K, there's little doubt that 32 volts over 7 mils, giving 4.6 K, indicates a 4.7 K nominal value resistor, whatever its actual precise value may be.

35. Here's another one. We measure 3.75 milliamperes of current and a voltage 5.65 volts across it. What is the resistance in the circuit?

36. Resistance in ohms equals volts divided by amps, or K ohms equals volts over milliamps, in this case, 5.65 over 3.75. A quick estimate gives you what probable value for the resistance? (1 K-ohm) (1.5 K-ohms) (2 K-ohms)

37. Yes. If we rounded 5.65 to 6, and 3.75 to 4, we'd get 1.5 exactly, and very quickly. If we rounded off 5.65 to 5.7 and 3.75 to 3.8, we'd calculate 1.5 exactly, but not as quickly. A calculator would divide 5.65 by 3.75 and get 1.506666. Who's right? It's really unimportant, since the in-fluence on the circuit by the meter itself may be greater than this; and anyway, most resistors are just not that precise.

Negative Numbers and Notations

1. In arithmetic you have worked with ordinary numbers without any qualification; that is, they were simple whole numbers or fractions. In electronics, you will need to work with plain rational and even irrational numbers, like $\sqrt{2}$ or π, imaginary numbers, like $\sqrt{-1}$, and negative numbers.

2. Negative numbers are easy to find; they have minus signs in front of them. In arithmetic the minus sign means a specific operation, subtraction; in algebra, the minus sign denotes a special kind of number. What is it? (irrational number) (positive number) (negative number)

3. Right. A negative number is useful because it implies an amount of one thing, in one of two directions. Minus five degrees centigrade means five degrees below the established point of zero degrees. You can see a thermometer scale has numbers above and below zero, increasing as they are further from zero.

4. Sometimes a horizontal number line is used to explain negative numbers, something like a thermometer scale rotated clockwise. At the right of zero, the positive numbers are one, two, and so on, to ten, or 100, while to the left of zero, the negative numbers get larger.

5. What is one way to describe a negative number? (a number for a quantity which can vary in two directions from 0) (a number which does not permit a positive event to be transacted)

6. Yes. Ordinary numbers like 2, 5, 5280, and $\frac{3}{5}$ often don't have a sign in front of them and are considered positive. They could be written +2, +5, +5280, and +$\frac{3}{5}$. Negative numbers like – 17, or "5 degrees below zero" always have the minus sign in front.

7. Negative numbers can be added to, subtracted from, multiplied, or divided by other negative or positive numbers. The operations and results are the same sort as with positive numbers, but you must be very careful how they are performed. For example, if you add a positive 5 and negative 3, you get positive 2. What do you think you would get as the sum of positive 4 and negative 8? (+12) (– 4) (– 12)

8. Yes. When you add a positive and a negative number, you just subtract the smaller from the larger, and give the result the sign, + or – , of the larger number. What is the sum of +12 and – 9? (+3) (– 3) (+21)

9. If you had an electrical resistor which had 3 amperes flowing through it in one direction from one leg of a circuit, and 2 amps in the opposite direction from another circuit leg, what

would be the net current through it? (1 amp) (2 amps) (3 amps)

10. Yes. Quantities with opposite signs or polarities are added by subtracting their so-called "absolute" values, that is, the unsigned numbers. Then what sign do you give to the result? (sign of the smallest number) (negative) (sign of the largest number)

11. Right. When adding several positive and negative numbers by hand, you must total the positive numbers, then separately total the negative numbers, then subtract the smaller total from the larger and give the result the sign of the larger total. If you use a small electronic calculator, of course, you don't have to do this; for addition, you just enter each number and then punch in its sign.

12. What is the sum of +3, a negative 5, +2 and – 4? (+3) (– 4) (14)

13. Yes. You already know that adding a +3 and a +4 is +7, and so on. If you add a – 1 to a – 5, you would get – 6. What would be the sum of – 8 and – 2? (– 6) (– 10) (+10)

14. Yes. To add numbers with the same sign, just add them and keep the same sign. To manually add positive and negative numbers, subtotal the positive, then the negative numbers, take the difference of the totals and assign it the sign of the largest subtotal.

15. You can add a lot of signed numbers at one time in that way; but, of course, subtraction involves only two numbers, the number being subtracted, and the number you're subtracting from. The rule for subtraction is "change the sign of the number being subtracted, then add." If you have a +7, and you subtract a – 2 from it, what would you get? (+2) (+7) (+9)

16. Right. You know that if you subtract a +3 from +10 you would get +7; when you subtract a – 2 from a minus 8 you would get – 6. What is the result of subtracting +4 from – 9? (+13) (– 5) (– 13)

17. If you add +17 and – 12, you'd get +5. Add +22 and – 18, and you'd get – 6. What would you get if you subtracted a +3 from – 11? (+14) (– 8) (– 14)

18. Yes. To add signed numbers, subtract the subtotals of positive and negative numbers, and use the sign of the larger subtotal on the result. To subtract two signed numbers, change the sign of the number being subtracted, and add, using the rule for adding signed numbers.

19. Multiplying and dividing positive and negative numbers is easy. You never need worry about which operation to perform, because to multiply, you just multiply, and to divide, just divide. The only matter to watch is the sign of the result, and here are the rules.

20. The result of multiplying or dividing two numbers with the same sign is always positive. The result of multiplying, or dividing two numbers with different signs is always negative. What is the product of +7 times negative 4? (+28) (– 28)

21. Right. What is the result of negative 8 divided by +2? (+4) (– 4)

22. Right. What is negative 12 divided by negative 4? (+3) (– 3)

23. Yes. In electronics you will need to work with algebraic quantities, and a review of procedures in algebra would be helpful. Much of the content of these Me programs is the same or similar to the Ma algebra programs. You may recall that because of positive and negative numbers, it is sometimes necessary in algebra to use different notation than you use in arithmetic.

24. For example, it is often convenient to show a division relationship as a ratio or fraction, with a horizontal bar and the divisor below it. For example, 12 divided by 17 could be shown as 12 seventeenths. How would we show the result of x plus y divided by 2? $(\frac{x}{2} + y)$ $(x + \frac{y}{2})$ $(\frac{x + y}{2})$

25. In electronics, which would you be the most likely to write? (5 amps $= \frac{10 \text{ volts}}{2 \text{ ohms}}$) (5 amps = 10 volts ÷ 2 ohms)

26. To show multiplication, you will usually avoid the x-shaped multiplication notation you used in arithmetic, since in algebra, x is often used for the variable, or an unknown value. To show multiplication, you can use a dot, as in x • y, for x times y, or you can just write xy, and the multiplication operation is assumed.

27. Unfortunately, you may wish to show that the numbers have signs, in which case you may use parentheses to indicate multiplication. Parentheses also may be used for grouping, with the group being multiplied by the value next to it. What is the result of 3 times the quantity in parentheses (4 – 2)? (12) (6) (– 18)

28. Yes. What is another good way of showing A times B? (A + B) (A x B) (AB)

29. Right. If E represents voltage, I, current, and R, electrical resistance, we can say that E = IR, or I times R. Transposing the formula slightly, we could also say that I equals E divided by R. How would we prefer to write it? $(I = \frac{E}{R})$ $(I = E \div R)$

30. Yes, in electronics we will usually show division in fractional form using a single horizontal bar. You learned about fractions in school, but you may have forgotten many of the rules. In the next program, you will review how to add and subtract fractions, by finding a

common denominator. Before you study it, you may wish to go over the 16 programs in the Mf fractions series.

31. Adding and subtracting fractions is more difficult, but right now, we will take a few minutes to remind you how to multiply and divide fractions, which is easier. What is $\frac{1}{2}$ times $\frac{1}{2}$? ($\frac{1}{4}$) ($\frac{1}{2}$) (1)

32. Yes, and $\frac{1}{2}$ times $\frac{1}{3}$ is $\frac{1}{6}$ ... $\frac{1}{2}$ times $\frac{2}{3}$ is $\frac{2}{6}$. To multiply fractions, you just multiply the denominators and numerators together. $\frac{3}{4}$ times $\frac{2}{3}$ is $\frac{6}{12}$. Sometimes after multiplication, the result can be reduced to lower terms. $\frac{2}{6}$ is $\frac{1}{3}$, and $\frac{6}{12}$ is $\frac{1}{2}$, of course. What is $\frac{3}{5}$ times $\frac{4}{7}$? ($\frac{3}{15}$) ($\frac{12}{35}$) ($\frac{34}{37}$)

33. Yes. To divide, which is the inverse of multiplication, you just invert the divisor fraction and then multiply as before. $\frac{3}{4}$ divided by $\frac{2}{3}$ would be the same as $\frac{3}{4}$ times $\frac{3}{2}$, which is $\frac{9}{8}$. What is $\frac{1}{2}$ divided by $\frac{2}{3}$? ($\frac{1}{3}$) ($\frac{3}{4}$) ($\frac{3}{5}$)

34. Yes. What is $\frac{15}{16}$ divided by $\frac{3}{4}$, reduced to lowest terms? ($\frac{5}{16}$) ($\frac{45}{64}$) ($\frac{5}{4}$)

35. Right. Of course $\frac{5}{4}$ is an "improper" fraction, that is, it's more than one, and could be expressed as $1\frac{1}{4}$. In electronics, we'll not worry as much as in arithmetic about impropriety. And if you wish to multiply a mixed number, like $2\frac{1}{2}$, by a fraction, say $\frac{2}{3}$, you'd have to convert it to an improper fraction, in this case $\frac{5}{2}$, before you multiplied numerators and denominators to get the product. What is the product of $2\frac{1}{2}$ times $\frac{2}{3}$? ($\frac{3}{5}$) ($\frac{4}{5}$) ($\frac{5}{3}$ or $1\frac{2}{3}$)

36. What is the result of $1\frac{3}{4}$ divided by $\frac{2}{5}$? ($\frac{35}{8}$ or $4\frac{3}{8}$) ($\frac{14}{20}$ or $\frac{7}{10}$) ($\frac{26}{20}$ or $1\frac{3}{10}$)

37. What is the product of $\frac{x}{2}$ times $\frac{y}{3}$? ($\frac{x+4}{6}$) ($\frac{xy}{6}$) ($\frac{3x}{2y}$)

38. What is the result of $\frac{A}{5}$ divided by $\frac{B}{4}$? ($\frac{4A}{5B}$) ($\frac{AB}{20}$) ($\frac{A+B}{20}$)

39. What is the product of $\frac{-x}{7}$ times $\frac{-y}{3}$? ($\frac{-xy}{21}$) ($\frac{xy}{21}$)

40. What is the result of $\frac{2x}{3}$ divided by $\frac{-5y}{7}$? ($\frac{14x}{15y}$) ($\frac{-14x}{15y}$) ($\frac{10xy}{21}$)

Adding and Subtracting Fractions

1. In learning arithmetic, the sums and differences of any two digits are easier to remember than their products and quotients, so addition and subtraction is taught before multiplication and division of whole numbers. But in working with fractions, addition and subtraction is more difficult than multiplication and division, so this entire electronics mathematics program is devoted to adding and subtracting fractions. In fact, four of the Mf series of programs are about this subject, and you may wish to review them.

2. What do you think is the sum of $\frac{1}{2}$ plus $\frac{1}{2}$? ($\frac{1}{4}$) ($\frac{1}{2}$) ($\frac{2}{2}$ or 1)

3. Right. "Like" fractions are fractions with the same denominator at the bottom. To add them, just add the numerators, at the top. What's the sum of $\frac{1}{5}$ and $\frac{2}{5}$? ($\frac{1}{5}$) ($\frac{2}{5}$) ($\frac{3}{5}$)

4. Right. To subtract "like" fractions, just subtract the numerator of the fraction to be subtracted from the first fraction. What's $\frac{3}{7}$ minus $\frac{2}{7}$? ($\frac{1}{7}$) ($\frac{5}{7}$) ($\frac{6}{7}$)

5. Yes. What's the sum of $\dfrac{3}{x^2 + 17xy^2 + 5z - 21b}$ plus $\dfrac{4}{x^2 + 17xy^2 + 5z - 21b}$? ($\dfrac{7}{x^2 + 17xy^2 + 5z - 21b}$) (can't be done)

6. Yes. When adding or subtracting "like" fractions, just add or subtract the numerators and place the result over the common denominator. Nothing to it, is there? Well, there is a catch, naturally. Most fractions you will deal with in electronics won't have the same denominator, so most of the work will be finding a common denominator and converting the fractions to it.

7. Let's say you wished to add $\frac{1}{2}$ and $\frac{1}{4}$. They're not like fractions. But if you changed $\frac{1}{2}$ to $\frac{2}{4}$, they would be. Then you could add numerators, and get $\frac{3}{4}$ as the sum of $\frac{1}{4}$ and $\frac{1}{2}$. What did you do to $\frac{1}{2}$ to change it to $\frac{2}{4}$? (added 2 to numerator and denominator) (multiplied numerator and denominator by 2)

8. Right. $\frac{1}{2}$ can be changed to $\frac{2}{4}$, or $\frac{3}{6}$, or $\frac{5}{10}$, by multiplying top and bottom of the fraction by the same number in each case. There's no change in the value of the fraction when you do this.

9. How would you change $\frac{3}{8}$ and $\frac{1}{4}$ into like fractions so they can be added? (multiply top and bottom of $\frac{1}{4}$ by 2; $\frac{1}{4} = \frac{2}{8}$) (add $\frac{1}{8}$ to $\frac{1}{4}$, get $\frac{3}{8}$, then invert; $\frac{1}{4} = \frac{8}{3}$)

10. Yes. If you were to add or subtract pairs of fractions like $\frac{1}{2}$ and $\frac{1}{4}$, $\frac{5}{6}$ and $\frac{7}{12}$, or $\frac{7}{8}$ and $\frac{5}{24}$,

only one of the fractions would have to be changed by multiplying the numerator and denominator by some number, since one denominator is a "factor" of the other. 2 is a factor of 4, 6 is a factor of 12, and 8 is a factor of 24.

11. What are the digital factors of 12? (2, 8, 4) (2, 6, 10) (2, 3, 4, 6)

12. Right. This doesn't count 1 and 12, of course. The factors of x^2 are x and x; the factors of AB are A and B, and the factors of 10 are 2 and 5.

13. Now let's say you needed to add $\frac{1}{3}$ ampere and $\frac{1}{4}$ ampere. What's the total current in amps? 3 can't be factored and the factors of 4 are 2 and 2, which is no help. One way to find a common denominator is to multiply, temporarily and off to one side, the denominators. This is 12 in our case. To convert $\frac{1}{3}$ to 12ths, we'd have to multiply numerator and denominator by 4, and get $\frac{4}{12}$. What would we do to convert $\frac{1}{4}$ to 12ths? (multiply top and bottom by 3, get $\frac{3}{12}$) (multiply top and bottom by 4, get $\frac{4}{16}$)

14. Yes. Then adding numerators, we'd say $\frac{1}{3}$ amp plus $\frac{1}{4}$ amp is $\frac{7}{12}$ amp. What's the common denominator you'd need to add $\frac{1}{2}$ and $\frac{1}{3}$? (3rds) (4ths) (6ths)

15. Right. Adding by this method is sometimes called "cross-multiplying," you may remember. For example, in adding $\frac{2}{3}$ and $\frac{5}{8}$, you multiply the numerator of the first fraction by the denominator of the second, then multiply the numerator of the second fraction times the denominator of the first, then add them over the product of the two denominators.

16. This means that to add $\frac{2}{3}$ and $\frac{5}{8}$, you multiply 2 by 8 and add 5 times 3, and set them above 3 times 8. What does this amount to? ($\frac{16+15}{24} = \frac{31}{24} = 1\frac{7}{24}$) ($\frac{16+5}{32} = \frac{21}{32}$)

17. Yes. To find the sum of $\frac{2}{5}$ and $\frac{3}{7}$, you'd have to cross-multiply 2 times 7, then add 3 times 5, all over 5 times 7. What would this give? ($\frac{29}{35}$) ($\frac{27}{35}$) ($\frac{35}{57}$)

18. What is the result of $\frac{5}{6}$ minus $\frac{1}{3}$? ($\frac{4}{6}$ or $\frac{2}{3}$) ($\frac{2}{6}$ or $\frac{1}{3}$) ($\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$ or $\frac{1}{2}$)

19. What is the result of subtracting $\frac{2}{3}$ from $1\frac{1}{6}$? Remember, $1\frac{1}{6}$ is the same as $\frac{7}{6}$, and $\frac{2}{3}$ equals $\frac{4}{6}$. ($\frac{2}{6}$ or $\frac{1}{3}$) ($\frac{3}{6}$ or $\frac{1}{2}$) ($\frac{4}{6}$ or $\frac{2}{3}$)

20. If you add 3 or more fractions, you may have to cross-multiply each numerator by more than one denominator. For example, what is the sum of $\frac{1}{2}$ plus $\frac{1}{3}$ plus $\frac{1}{5}$? The lowest common denominator is 2 times 3 times 5. This requires you to multiply the numerator, 1, of the fraction $\frac{1}{2}$ by both 3 and 5, the numerator of the fraction $\frac{1}{3}$ by both 2 and 5, and the numerator of $\frac{1}{5}$ by both 2 and 3. This gives 15, plus 10, plus 6 as a numerator over the denominator, 30. What's the sum of these fractions? ($\frac{15+10+6}{30} = \frac{31}{30} = 1\frac{1}{30}$) ($\frac{15+10+6}{30} = \frac{26}{30} = \frac{13}{15}$)

21. What is the sum of $\frac{1}{4}$ plus $\frac{1}{3}$ plus $\frac{1}{6}$? In this case, a common denominator could be 4 times 3 times 6, and we could cross-multiply each numerator by the other two denominators. But by looking at them, we see that 4 times 3 is 12, and 6 is a factor of 12, so we could use 12 instead of 72 as the denominator. In fact, we could call 12 by a special name in this case. What is it? (Baker's Dozen) (Round Lot) (Lowest Common Denominator)

22. Right. In any case, we are always multiplying both the numerator and denominator of each fraction by the same number, so it is not changed in value, before it is added or subtracted. Cross-multiplication is one way of doing this. When it doesn't provide the lowest common denominator, however, and you wish to find it, you may look at the factors of each denominator to determine the lowest common denominator.

23. How would you add $\frac{5}{6}$ and $\frac{3}{8}$? The product of the denominators is 48, but the lowest common denominator is 24, so you could multiply $\frac{5}{6}$, top and bottom by 4, and $\frac{3}{8}$, top and bottom by 3. What would this give you as their sum? ($\frac{29}{24} = 1\frac{5}{24}$) ($\frac{11}{24}$)

24. In the case of $\frac{5}{6}$ and $\frac{3}{8}$ it was hardly worth a lot of trouble to worry about the lowest common denominator. If you can see a common denominator lower than the product of the two or more denominators, it might be helpful in order to avoid big numbers.

25. In the same way that we don't change a fraction by multiplying numerator and denominator by the same number, we don't change it, as you may recall, by dividing by the same number. You know that $\frac{2}{4}$ is $\frac{1}{2}$, and $\frac{3}{6}$ is also $\frac{1}{2}$. $\frac{50}{100}$ is $\frac{1}{2}$, too. $\frac{25}{100}$ is $\frac{1}{4}$, and so on. What is $\frac{9}{27}$? ($\frac{1}{4}$) ($\frac{1}{3}$) ($\frac{1}{2}$)

26. What is $\frac{24}{16}$, reduced to lowest terms? ($\frac{18}{16}$) ($\frac{3}{2}$ or $1\frac{1}{2}$) ($\frac{12}{8}$)

27. What is the sum of $1\frac{3}{8}$ and $\frac{5}{6}$? ($1\frac{3}{8} = \frac{11}{8} = \frac{33}{24}$; $\frac{33}{24} + \frac{20}{24} = \frac{53}{24} = 2\frac{5}{24}$) ($1\frac{3}{8} = \frac{11}{8} = \frac{44}{24}$; $\frac{44}{24} + \frac{18}{24} = \frac{62}{24} = 2\frac{7}{12}$)

28. What is $\frac{3}{8}$ minus $\frac{4}{5}$? In this case we cross-multiply, and then subtract the resulting numerators. ($\frac{35 - 32}{40} = \frac{3}{40}$) ($\frac{15 - 32}{40} = -\frac{17}{40}$)

29. Right, we get a negative number in this case. What is the sum of $\frac{x}{24}$ plus $\frac{y}{24}$? ($\frac{xy}{24}$) ($\frac{x + y}{24}$)

30. Right. What is the sum of $\frac{a}{2} + \frac{b}{3}$? ($\frac{3a + 2b}{6}$) ($\frac{a + b}{6}$) ($\frac{a + b}{33}$)

31. What is the sum of $\frac{1}{3} + \frac{3}{4} + \frac{2}{5}$? ($\frac{1(4)(5) + 3(3)(5) + 2(3)(4)}{} = \frac{20 + 45 + 24}{60} = 1\frac{29}{60}$) ($\frac{1(4)(5) + 3(3)(5) + 2(3)(5)}{(3)(4)(5)} = \frac{20 + 30 + 35}{60} = \frac{(3)(4)(5)}{} = 1\frac{17}{60}$)

32. What is a way of expressing the rules for adding fractions? (always get the lowest common denominator, then multiply numerators together, and multiply denominators) (find common denominator, convert fractions to it, add numerators)

33. What is the common denominator to use for subtracting $\frac{2}{3}$ from $\frac{5}{7}$? (10) (21) (25)

34. What is $\frac{5}{7}$ less $\frac{2}{3}$? ($\frac{15}{21} - \frac{14}{21} = \frac{1}{21}$) ($\frac{15}{21} - \frac{17}{21} = -\frac{2}{21}$)

35. What is a way for expressing the rule for subtracting fractions? (convert fractions to common denominator, subtract numerators) (cross-multiply the common factor of the negative numerator; add denominators and carry the remainder)

36. What can you do to a fraction without changing its value? (multiply or divide numerator and denominator by the same number) (add or subtract the same number to numerator and denominator)

37. In the previous program, you learned how to multiply and divide fractions. What's the rule for multiplying fractions? (add numerators and add denominators) (multiply numerators and multiply denominators)

38. Right. What's the rule for dividing fractions? (invert divisor, multiply fractions) (find the lowest common denominator, then subtract numerators)

39. What is $\frac{3}{5}$ divided by $\frac{5}{6}$? ($\frac{35}{30} = 1\frac{1}{6}$) ($\frac{18}{25}$)

40. Many persons find it difficult to remember how to work with fractions. If you do, repeat these programs, and study other materials, such as the Mf programs. Good luck!

Roots and Powers

1. When a number like 3 is multiplied by itself, the product is called the "square" of 3. This is probably because two numbers are often multiplied to give the area of a surface. For example, there are 3 feet in a yard and 9 square feet in a square yard.

2. In turn, we say that 3 is the "square root" of 9. "Root" means the basic part of something. In algebra, "root" sometimes means the basic result of an equation when solving for an unknown. The "square root" of a number means the number which, when multiplied by itself, gives the original number.

3. This sign is called the "radical sign." It refers to the base number which, when multiplied by itself a given number of times, gives the number under the sign. The most commonly used root is the square root. When the radical sign is used without the small exponent-sized number, it means "square root." A small 3 by the notch would indicate the cube root or the number which multiplied by itself twice gives the number under the sign, and so on.

4. What would you think this expression means? $\sqrt[4]{16}$ [16 divided by 4 (4)] [the fourth root of 16 (2)]

5. In other programs you have seen some electrical formulas that use squares and square roots. In this program you will learn a quick way to find squares and square roots in your head. Quickly, now, what is the square of 14? (144) (196) (288)

6. Yes. 196. You should have guessed that quickly, because if you remember your multiplication table up to 12 times 12 is 144, you could have guessed it was more than 144, and probably smaller than 288. You should also recall that 11 squared is 121. If you don't remember the multiplication table up to 12 times 12, go back and practice. Program Mn 5 will help.

7. Next, memorize the squares of these eight numbers. Look at them carefully. $13^2 = 169$; $14^2 = 196$; $15^2 = 225$; $16^2 = 256$; $18^2 = 324$; $22^2 = 484$; $25^2 = 625$; $27^2 = 729$. Notice that 13 squared and 14 squared are similar—169 and 196. 15 squared should be easy; it's 225 and 25 squared is 625. What is 22 squared? (242) (484) (848)

8. What are 13 squared, and 14 squared, respectively? (169, 196) (196, 169) (131, 242)

9. Yes. What are 15 squared and 25 squared, respectively? (625, 225) (225, 625) (522, 526)

10. What is 22 squared? (121) (242) (484)

11. Right. What is 30 squared? (300) (600) (900)

12. Right. What is 27 squared? (625) (729) (969)

13. What is 18 squared? (256) (324) (484)

14. Yes. What is the square root of 324? (16) (18) (20)

15. What is the square of 16? (216) (256) (296)

16. What is the square root of 729? (27) (29) (30)

17. Yes. What are the squares of 22, 25, and 27, respectively? (484, 625, 729) (424, 625, 727)

18. What is the square of 20? (200) (400) (4,000)

19. Right. What is the square root of 225? (15) (25)

20. Here are the 8 numbers again. 13, 14, 15, 16, 18, 22, 25, 27. Which answer lists their squares correctly? (169, 196, 225, 296, 324, 484, 625, 727) (169, 196, 225, 256, 324, 484, 625, 729)

21. What whole number is close to the square root of 530? First try to recall the square roots of 484 and 625. 530 is closer to 484 than 625. (22) (23) (25)

22. You can quickly determine the squares of most numbers. First, round them off to one or two digits, then multiply them by themselves mentally, on paper, or with a small calculator. 11.99 squared is 144 for all practical purposes, for example. But getting square roots is a little harder. Let's try the following.

23. To find the approximate value of a number's square root, first round the number off to the nearest 2 or 3 places. You may either add or eliminate 2, 4, 6, or any even number of zero digits. For example, round off 7 to 700 times 10^{-2}, and 4,783 to 48 times 10^2. How would you round off 876? (87×10^2) (88×10^2) (876)

24. Right, it's already a three digit number. After practice, you will be able to estimate closely the value of any two- or three-digit number. For our purposes, how would you round off 17.6? (17×10^2) (18) (18×10^2)

25. Correct. How would you round off 6,573,291? (657×10^4) (657×10^2) (657)

26. Yes. And you would round off 10,421 to 104 times 10^2. How would you round off 53? (52) (53) (5300 x 10^{-2})

27. Yes, it's already a two-digit number. Now, after rounding off your starting number to a two- or three-digit number, estimate its square root. The smallest two-digit number is 10, and its square root is about 3.2, or more precisely, 3.16. The largest three-digit number is 999 and its square root, of course is about 32, or more precisely, 31.6. The largest two-digit number is 99, and its square root is 9.95; the smallest three-digit number is 100, and its square root is 10.

28. The square of 7 is 49. The square of 8 is 64. Now guess what the approximate value of the square root of 50 is. (6.9) (7.1) (7.9)

29. Yes, of course. Now, what is the square root of 7? First, round off to 700 times 10^{-2}. Remember that the square roots of 625 and 729 are 25 and 27. What is the square root of 700? (25.5) (26.5) (27.5)

30. Yes, that is a pretty close approximation. If you estimate the square root of 700 at 26½, what is the square root of 7? (.265) (2.65) (26.5)

31. Yes. We said that by rounding off 6,573,291 we'd get 657 times 10^4. What's the square root of 657? (25.5) (26.5) (27.5)

32. Correct. When you add back the zeros to the square root, you only add half as many places. Now, what's the approximate square root of 6,573,291? (2,550) (2,650) (25,500)

33. Yes. We rounded off 4,763 to 48 times 10^2. What's the square root of 4,763? (61) (69) (71)

34. Yes, very close. Of course you remember the square roots of 25 and 36. What's the square root of 28? (5.0) (5.3) (5.8)

35. What's the square root of 876 and the perfect squares near it? [29.6 (729)(900)] [26.7 (625)(900)] [32.6 (777)(900)]

36. Yes. A few minutes ago we rounded 17.6 to 18. 18 is between 16 and 25, so its square root is between 4 and 5. What would you guess it is? (4.2) (4.5) (4.9)

37. 10,421 rounds to 104 x 10^2. You recall the square root of 100 is 10. What's the square root of 10,421? (10.2) (100) (102)

38. Right. Don't forget, round off the number to a two - or three - digit number, times an even power of ten if necessary. Then estimate the square root by recalling the perfect squares you memorized, interpolating mentally, then add back half of the places you set aside. You will need to repeat this program several times, then practice this procedure occasionally on random numbers.

39. A good number to remember is the square root of 2, which is 1.414, or, for electronics, we'll say 1.4. Also, the square root of ½ or .5 is .707. This makes the square root of 200 equal to 14.14 and the square root of 50, 7.07. What's the square root of 20,000? (14.14) (141.4) (1, 414)

40. Yes. The square root of 10,000 is 100; and the square root of 1,000,000 is 1,000, of course. What's the square root of 10? (1.0) (3.16) (5.0)

41. You just learned that the respective square roots of .5 and 50 are .707 and 7.07. What are the square roots of 5 and 500, respectively? (2.24, 22.4) (3.5, 35) (4.7, 47)

42. Yes. You might have recalled that 22 squared was 484. Estimating square roots is quite easy to do and quickly learned, if you're willing to spend an hour in intensive practice. For most electronic purposes you can do it in your head faster than you can grab a slide rule, or a small electronic calculator, so don't be a slave to a gadget. Good Luck.

1. Even though many values in electronics are approximate, you will, unfortunately, have to work with both large, unwieldy numbers and very small decimal fractions. You may easily make mistakes if you are not careful in placing the decimal point or in counting the zeros to the left or right of it.

2. For example, you may remember that a farad is a unit of capacitance which stores a coulomb of charge at one volt. But most capacitors are rated in microfarads or picofarads, which are, respectively, millionths and trillionths of a farad. Sometimes we deal with resistances in ohms, but at other times in millions of ohms, or megohms. Frequencies are sometimes in hertz, or cycles per second, but at other times are in kilocycles, megacycles, or even gigacycles (or gigahertz).

3. If you, as an electronics technician or technologist, must work with numbers like this, you'll find it necessary to think in terms of "powers of 10." This means that you will work with our decimal base, which is ten, raised to some power by the use of an exponent. Which number is the exponent in the expression 3.14×10^8? (3.14) (10) (8)

4. Right. As you may recall, 10 "squared" is 10 times 10, ten "cubed" is $10 \times 10 \times 10$, 10 "to the fourth power" is $10 \times 10 \times 10 \times 10$, and so on. In such an expression, the base number appears the same number of times as the exponent. You should note, however, that the multiplication operations occur one less time than the exponent. 10 to the fifth power means 10 may be written 5 times, but there are only 4 multiplication operations.

5. What does 10^8 mean? $(10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10)$ $(10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10)$ $(10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10)$

6. When you multiply a number, say x, by itself, the exponent is two and the number is called x^2. Multiply by x again and you get x cubed. If you took 10^3 and divided it by 10, you'd have 10^2, and division of 10^2 by 10 would just give you 10. We write 10 without an exponent because the exponent is understood. What is the understood exponent? (0) (1) (2)

7. Yes. Any number, written without an exponent, can be considered to have the exponent 1. If you divided x, or x to the first power, by x, you'd get x to the 0 power. What is x to the zero power? $(\frac{x}{0})$ $(\frac{0}{x})$ (1)

8. Right, one. One divided by any number is that number to the minus one power, or its

inverse. Another term for this, you may recall is reciprocal. What is 10 to the minus one power? (1) (.1) (.01)

9. Yes. And ten to the minus two power is one one-hundredth. What is 10 to the minus three power? [$\frac{1}{1000}$(.001)] [$\frac{1}{10,000}$ (.0001)] [$\frac{1}{100,000}$(.00001)]

10. Yes. If you were to calculate the power used by a 4.75-ohm resistor connected across 1250 volts, you would get approximately 328,947.36 watts. Scientists and technologists try to avoid misleading implications of excessive precision or significance by rounding off numbers. To do this, they use what is called "scientific notation." This consists of a number, usually between 1 and 10, which has only as many digits as necessary, times some power of 10. How would you write the watts used by the resistor in scientific notation? (3.28×10^5 W) (328×10^3 W) (328,947.36 W)

11. How else would you write 3.2 x 10 to the third power? (32,000) (3,200) (320)

12. What system uses a number from one to 10, times a power of 10, to indicate large values? (technical notation) (engineering notices) (scientific notation)

13. Right. Scientific notation is used for very small values, too. If you wished to indicate the power used by a one-megohm resistor connected across one-tenth of a volt, it would be one one-hundred-millionth of a watt. Since one thousandth is point zero zero one and one millionth is .000001, a hundred millionth is .00000001. One millionth is 10^{-6}. What would one-hundred-millionth be as a power of 10? (10^{-7}) (10^{-8}) (10^{-9})

14. A microampere is one-millionth of an ampere. How would you write it as a decimal and as a power of 10? (.000001 amp; 10^{-6} amp) (0.00001 amp; 10^{-5} amp) (.0001 amp; 10^{-5})

15. Yes. Let's say the voltage across a megohm resistor was one-fifth of a volt. The power produced is 4 hundred millionths of a watt. How would you write it? (4.0×10^{-7}) (4.0×10^{-8}) (4.0×10^{-9})

16. In order to convert numbers involving decimal fractions into scientific notation without changing their value, you must change the exponent of ten by one for each zero or other digit past which you move the decimal point.

17. For practice, let us consider the shift of decimal points with changes in the exponent of ten. This number is 270 ten-thousandths. It can also be shown as 27 times 10 to the minus four, or 2.7 times 10 to the minus three.

18. All these numbers, for example, have the same value. Study these to understand why. .00068; .068 x 10^{-2}; 6.8 x 10^{-4}.

19. How would you express 9,876,543 in scientific notation with only three significant digits? (9.88×10^4) (9.87×10^6) (9.88×10^6)

20. Here is a table of decimal numbers and powers of ten. (Refer to filmstrip.)

21. How would you express 125 microamps (which is 125 millionths of an ampere)? $(125 \times 10^{-6}$ amp) $(1.25 \times 10^{-4}$ amp) (either is correct)

22. Yes. In multiplying numbers which have been raised to a power, add the exponents. For example, 10 squared times ten cubed equals 10 to the fifth power. In the case of negative exponents, the exponents, of course, are added algebraically. We would say that x to the fifth power times x to the minus second power is x cubed. What would you say is the product of 2 times 10 to the sixth power times 3 times 10 to the minus sixth power? (6) (60) (600)

23. From this, you can perhaps see that to "square" a number which has been raised to a power, you simply double the exponent. To obtain the square root, you half the exponent. You must of course also take the square root of the stem number, as well as of the power of ten. What is the square root of one million, or 10^6? (10^2) (10^3) (10^4)

24. What is the square root of 4×10^4? $[2 \times 10^2 \ (200)]$ $[4 \times 10^2 \ (400)]$ $[2 \times 10^4 \ (20,000)]$

25. Sometimes the exponent of 10 is an odd number. In this case, when you half the exponent to obtain the square root, you would get an extra multiplier of 3.16, which is the square root of 10. For example, the square root of 10 million, or 10^7, is $10^{3\frac{1}{2}}$. $10^{3\frac{1}{2}}$ equals the square root of 10 times 10^3, or 3.16 times 10^3. The square root of 40 million, or 4 times 10^7, is 2 times $10^{3\frac{1}{2}}$, or 6.32 times 10^3. Could this be written as 6,320? (Yes) (No)

26. What is the square of 70,000, or $(7 \times 10^4)^2$? $[49 \times 10^8; 4.9 \times 10^9; (4,900,000,000)]$ $[49 \times 10^7; 4.9 \times 10^8; (490,000,000)]$ $[70 \times 10^7; 7 \times 10^8; (700,000,000)]$

27. Yes. What is the square root of 1.6×10^3? $(\sqrt{1.6 \times 10^3} = 4 \times \sqrt{10} = 12.64)$ $(\sqrt{1.6 \times 10^3} = \sqrt{16 \times 10^2} = 4\sqrt{10^2} = 40)$

28. Correct. Suppose we have a 27 K, or 27,000 ohm, resistor with 2 milliamperes flowing through it. What is the voltage across it? A milliampere is one-thousandth of an ampere, so we write the current as: point zero zero two amperes, or 2 times 10^{-3}. 27 K or 27 kilohms, or 27,000 ohms, is 27 times 10^3. We could also write it as 2.7 times 10^4, but we will not do so in this case for what will become very obvious reasons.

29. What is the product of 2 times 10^{-3} amps x 27 x 10^3 ohms? (5.4 volts) (54 volts) $(54 \times 10^2$ volts)

30. Yes. Electronics technicians sometimes remember that when current in microamps is multiplied times resistance in megohms, or when current in milliamps is multiplied times resistance in kilohms, the result amounts to the same voltage as if the current and resistance were in amps and ohms. Unfortunately, however, the potential for mistakes due to the loss of a decimal place or two is so great that we cannot use short cuts except with great care.

31. The power in a resistor equals the voltage across it times the current through it. How much power is used in a resistor across 1000 volts, with one milliamp of current flowing? (10^{-3} watt; .001 watt) (1 watt) (10^3 watts; 1000 watts)

32. The power in a resistor also equals the current through it, squared, times the resistance. How much power is used in a 100,000-ohm resistor, carrying one milliampere? [$(10^{-3})^2(10^5)=10^0=1$ watt] [$(10^{-3})^2(10^5)=(10^{-6})(10^5)=10^{-1}=.1$ watt]

33. Yes. Voltage is resistance times current. What is the voltage across a 100,000-ohm resistor carrying one milliamp? [$(10^5)(10^{-3})=10^2=100$ V] [$(10^5)(10^{-3})=.01$ V $=10^{-2}$ V]

34. Right. You have been studying what are fundamentally resistor and power problems, but in many cases, the most extreme values occur in capacitors. You may recall that we said that capacitors are rated in farads, microfarads, and picofarads. Microfarads are millionths of a farad and picofarads are millionths of a microfarad. How would you express a picofarad in farads? [10^{-3} (thousandth)] [10^{-6} (millionth)] [10^{-12} (trillionth)]

35. Yes. Remember, when multiplying powers of the same number, add the exponents and when dividing, subtract them. For example, current equals voltage divided by resistance. What is the current due to 10 volts across a 1000-ohm resistor? ($10^1 \div 10^3 = 10^{-2} = .01$ amp or 10 milliamps) ($10^1 \div 10^3 = 10^2 = 100$ amps)

36. Yes. Also, when squaring a number raised to a power, double the exponent. In effect this merely means adding the exponents of the two multipliers. Squaring 10^3 is multiplying 10^3 by 10^3, to get 10^6, by adding exponents.

37. On the other hand, if you want to find the power of a power, you calculate it by taking the product of the two exponents. For example, what does 10^9 raised to the 3rd power equal? (10^6) (10^{12}) (10^{27})

38. What is the square root of 9 x 10^{-6}? ($\sqrt{9 \times 10^{-6}} = 3\sqrt{10^{-6}} = 3 \times 10^{-3} = .003$) ($\sqrt{9 \times 10^{-6}} = 3\sqrt{10^{-6}} = 3 \times 10^{-12}$)

39. What is 10 to the fifth power squared, divided by 10 to the sixth power? (10^4) (10^6) (10^8)

40. You may need a considerable amount of practice on scientific notation using powers of ten. A suitable calculator can help a lot. No matter whether you use a calculator, slide rule, or your own brain, the concept and procedures for powers of ten are very important to those who work in electronics, so you should repeat this program. Study it carefully.

Equations and Formulas

1. In the first eight programs of this series, you have learned to work with quantities of various kinds, and to perform certain operations with whole numbers and fractions. In the next 6 programs, you will learn how to deal with relationships between quantities, some of which are very complex. The field which covers these relationships is called algebra, and the most useful device in algebra is the equation, which is an encoded statement describing two quantities, which are said to be equal.

2. How might you describe an equation? (a band around the earth) (statement that two quantities are equal)

3. Yes. Equations are written like this, with the left side, or member, being stated as equal to the right side, or right member. The mathematical expressions on each side may be made up of one or more numbers, letters, or other symbols which are shown to be added together, subtracted, multiplied, or divided.

4. Is this an equation, or just a complex mathematical expression? $[4x^2 - \frac{17x}{4} + 22 = 10]$ (equation) (expression)

5. Yes. An equation, of course, is not a problem. It is merely a statement involving a relationship of known quantities with one or more unknown quantities, represented by letters, which has two equal sides or members, and which, presumably, is helpful in solving a problem. There may be a solution to an equation, and it, too, may help solve the problem.

6. Let's say that a can of paint, plus a 5 pound weight, equals 15 pounds. Which of these is the solution of the equation, $P + 5 = 15$? $(P = 10)$ $(P = P)$ (15 lbs.)

7. Yes, the statement that the unknown, in this case P, the weight of the paint, equals some specific quantity, is called the "solution." The value of P which "satisfies," or makes the original equation an identity, is called the "root" of the equation. What is the "root" of the equation $P + 5 = 15$? (5) (10) (15)

8. Yes. Let's review a bit. What is the "solution" of $P + 5 = 15$? (10) $(P = 10)$ $(P + 5)$

9. Yes, and "10" is the root. You can easily understand the basis for the word "equation" as meaning an equality, which is a statement about different but equal quantities, or two things which have been equated. There are also statements called "inequalities," which declare that one quantity is larger or smaller than another. For example, you might say that $P + 5$ is smaller than 16 this way. $P + 5 < 16$

10. What kind of algebraic statement is $P + 5 < 16$? (equation) (inequality)

11. Right. Returning to our study of equations, we need to mention some helpful rules, which sometimes seem obvious, that we call axioms. For example, if $P + 5 = 15$, then any ordinary arithmetic operation which we do to the left side, $P + 5$, we can do to the right side, or 15, and still maintain the equality. We can add the same number to both sides, subtract the same number, multiply both sides by the same number, or divide by the same number, and the sides remain equal.

12. Of course you must be sure to perform the arithmetic operation to both sides of the equation, and to the entire expression on each side. If you added 2 to each side of $P + 5 = 15$, what would the equation say? $(2P + 5 = 17)$ $(P + 5 + 2 = 17$ or $P + 7 = 17)$

13. Right. How would $P + 5 = 15$ look after you multiplied both sides by 2? $(2P + 10 = 30)$ $(2P + 10 = 15)$

14. Yes. What would $P + 5 = 15$ be after dividing both sides by 5? $(\frac{1}{5} P + 1 = 3)$ $(P + 5 = 3)$

15. What would it be after subtracting 5 from both sides? $(P = 10)$ $(5P - 5 = 5)$

16. We have used the letter "P" to stand for the weight of a 10-pound can of paint. We could have used X, or Y, or Z, or some other English letter, or Greek letter, or symbol. In electronics, and in general mathematics, it is customary to use some symbols frequently for certain quantities, so you must be observant to note what they are used for in equations and formulas. For example, π is universally used to denote the ratio between a circle's circumference and its diameter; E is often used for voltage, I for current, and R for resistance. What is X often used for? (mystery) (approval) (unknown quantity; also multiplication)

17. Right. In formulas and equations, we will generally avoid using an X for the multiplication operation sign, and use letters and numbers placed adjacent to each other for multiplication, and over and under a bar, for division. Occasionally, we may use parentheses to show the application of a multiplier to two or more terms which are added together.

18. Which formula would you probably use to show that voltage equals current times resistance? $(E = IR)$ $(E = I \times R)$

19. Yes. A formula is a rule, written as an equation, which expresses some general relationship. Most equations, of course, are created for special situations to solve some problem, but they may make use of a formula for the general rule. Is a formula an equation? (Yes) (No)

20. Yes, although not all special equations are formulas. A "term" is a combination of numbers or letters by multiplication. $2\pi X$ is a term. $Y + 3$ is not a single term. Which of these is a "term"? $(X - 2)$ $(3XY)$ $(2A + B)$

21. Yes. 3XY is a term which indicates that 3, X, and Y have been multiplied together. $\frac{X}{2}$ or $\frac{2}{X}$ are also single terms, combined by division. They might also be considered as multiplied terms, since $\frac{X}{2}$ is merely X times $\frac{1}{2}$.

22. When terms are combined by addition or subtraction they are called "polynomials." An expression which contains three terms added together is called a "trinomial." What would you call two terms added together? (binomial) (duonomial) (tunomial)

23. What is $3x^2y + 5y^2$? (a term) (a binomial) (a trinomial)

24. As you may have gathered or assumed, an "expression" is a collection of numbers or symbols combined by any operation and may include one or more terms, or even more than one polynomial. Would you say this is an expression? $\dfrac{(x + 3)(y^2 - 24 + 7)}{(x^2 + 5)}$ (Yes) (No)

25. Adding two terms which are "unlike," that is, which contain different unknowns, or the same unknowns at different powers, is quite simple. You just note the addition and that's all you do. $4x + 2y$ is just $4x + 2y$ for as long as x and y are unknown. $3x^2$ and $5x$ must be written $3x^2 + 5x$. Adding $4a + 2b + 6$ must stop with a trinomial. But can you combine $7xy$ and $23xy$ by addition into one term? (Yes, 30xy) (No, 7xy + 23xy)

26. What's the simplest way of writing $2x^2z$ after subtracting $7x^2z$? $(2x^2z - 7x^2z)$ $(-5x^2z)$

27. Think carefully, now. What is the product of $3xy$ times $-2z$? $(-6x^2y)$ (x^2y) $(3xy - 2x)$

28. Yes. What is the sum of $5xy$ and $-4x$? $(5xy - 4x)$ $(xy - x)$ $(-20x^2y)$

29. Yes, the two terms were not "like" terms, so they couldn't be combined, by addition, into one term. Which of these are "like" terms? $[4a + 6b - 5c + 2ab - 3b + 17bc + \frac{b}{3}]$ (4a + 2b, and 5c + 17bc) (6b, - 3b and $\frac{1}{3}$ b or $\frac{b}{3}$)

30. Right. One way to add polynomials is to arrange their terms in columns of like terms, and take their algebraic sum. To add these polynomials $(a + 3b - 2c, 2b - 6a + 5c, 4c + 7a - 8b)$ arrange the a's, b's and c's in columns and add their coefficients, the numbers multiplied by them. What is their sum? $(2a + 10c)$ $(3a - 7b + 8c)$

31. When one term or polynomial is to be subtracted from another, it is probably best to write it down first, then write it again, but with its signs reversed, below the expression from which it is to be subtracted. Then proceed with the algebraic addition, which, of course, is either an arithmetic addition or subtraction. You should review program five about this.

32. For example, let's write $7a - 3b + 2c$, and subtract from it, $5a - 4b + 7c$. When we write the second expression below the first we change its signs to get $-5a + 4b - 7c$. Then adding algebraically, we can use the signs of the terms as indicators of arithmetic operations.

33. $7a$ and $-5a$ add to give $2a$; $-3b$ and $-4b$ give $-7b$; $+2c$ and $+7c$ give $+9c$. What is the resulting difference of the original expressions? $(2a - 7b + 9c)$ $(2a - 4b + 7c)$

34. Multiplication of single terms with polynomials is merely multiplying each term of the polynomial by the multiplier. The term $2x$ times the polynomial $3a + 5b$ just gives $6ax + 10bx$.

What is 3y times 4c + 6d? (12cy + 24cd + 18y) (12cy + 18dy)

35. We can divide a polynomial by a single term by dividing each term in the expression. 5ax + 15bx – 25cx, divided by 5x, gives a + 3b – 5c. What is 6a – 10b + 12 divided by 2? (3a – 5b + 6) (3a – 10b + 14)

36. Polynomials can be multiplied and divided by other polynomials, too. a + b times x + y is just ax + ay + bx + by, since each term in the multiplicand must be multiplied by each term in the multiplier. This is somewhat like multiplying 23 by 45, where each digit must be multiplied by each digit. But in the case of decimal-base numbers they can be combined, and while unknowns remain unknown in value, unlike terms must remain separated.

37. What is the product of 2c + 3d times x + y? (2cx + 2cy + 3dx + 3dy) (5cxy + 5dxy)

38. Yes. Sometimes, however, you will be able to combine terms. What is the product of x + y, and 2x + 3y? ($2x^2 + 2xy + 3xy + 3y^2$ or $2x^2 + 5xy + 3y^2$) ($2x^2 + 2x + 5xy + 3y + 3y^2$)

39. If you find it convenient, you may place all like terms in the same column. For instance, when multiplying $x^2 + 2xy + y^2$ by x – y, we get some x^3 and y^2 terms, and two terms each with x^2y and xy^2. Are these terms arranged this way? (Yes) (No)

40. In performing divisions of polynomials by single terms, you can see by a quick inspection if each of the polynomial terms can be divided evenly by the divisor terms. If they can't, the division must remain just an indicated operation.

41. When dividing a polynomial by another polynomial, you will need to learn to check by inspection or trial if it can be done evenly. This is called "factoring." For example, $x^2 + 2xy + y^2$ can be divided evenly only by x + y, since it is the product of x + y times itself. And $x^2 – y^2$ is the product of x + y and x – y, since the center terms cancel out in the addition, when cross-multiplying. These common types of expressions will become more familiar to you with practice. What is $x^2 + 2x + 3xy$ divided by x? ($x^2 + x + 2xy$) (x + 2 + 3y)

42. You have noticed that parentheses or brackets have been used to group together the terms of complex expressions which must be treated as a whole for multiplication or division. And this is true of subtraction, too, until signs are changed in the process of the operation. For example, in 2 times 5 minus 3, it is very important to know if two is being multiplied by 5, or by the quantity 5 minus 3. If the 5 and 3 form a single quantity, as indicated by parentheses, then the indicated operation within the parentheses is done first. If there aren't parentheses, the multiplication or division operations are performed first in the order in which they occur.

43. A review of these principles will be helpful before you go on. In the next program you will learn to perform operations with unknowns and solve problems with equations.

MATHEMATICS FOR ELECTRONICS   Me 10
Operations with Unknowns

1. Let's assume you have an electric circuit which has a circuit-breaker which interrupts the power when 20 amperes or more is drawn from its circuit by a toaster and a lamp. You slowly increase the current to a lamp by an adjustable control until it is 6 amperes, and the breaker operates, cutting off the current to both the toaster and the lamp. What was the toaster current, X? ($6X = 20$, divide by 6, $X = 3\frac{1}{2}$ amps) ($X + 6 = 20$, subtract 6, $X = 14$ amps)

2. Right, the sum of the two currents was 20 amps when the breaker operated, so the toaster current was 14 amps. You could probably set up the relationship in your head and solve it without any formal use of equations and algebra. But there will always be relationships so complex, or so many relationships which must be used by other persons, or programed into a computer, that the techniques of algebra will become useful, and sometimes essential.

3. Practice in changing verbal statements into algebraic expressions and equations will help you solve the types of problems which are typical of electronics. For example, what number, when increased by six, is 20? Write $X + 6 = 20$. You can quickly see that subtracting 6 from both sides of the operation gives the solution.

4. Five similar toasters draw a total of 100 amperes. Five times what number equals 100? We write $5X = 100$. What do you do to find the solution of the equation? (Divide both sides by 5) (Add 5 to both sides)

5. Yes. In algebra we assume that since one side of an equation equals the other, we can perform identical operations on each side without changing the truth that the two sides are equal. The values of each of the sides are changed, of course, but their equivalence has not.

6. Can we add or subtract the same amount to or from both sides of an equation without changing it from a true equation? (Yes) (No)

7. Can we multiply both sides of an equation or divide by the same number without changing it from a true equation? (Yes) (No)

8. Yes, but we must be sure that we add, subtract, multiply, or divide both sides properly, making certain that the operation is performed to the entire expression on each side. By the way, do you think you could square, or take the square root of, each side of an equation without unbalancing it? (Yes) (No)

9. Equations which can be solved by a single arithmetic operation are almost unnecessary. For example, if you ask what is the maximum current each toaster may draw if five of them are connected to a 75-ampere circuit, the single operation of dividing is so apparent that you might not need to write $5X = 75$.

10. And if you wished to know the total current if one-fifth of it equals 15, you would hardly need to write $\frac{1}{5}X = 15$, which can be solved by dividing each side by $\frac{1}{5}$. But if you must perform two or more operations, the relation often gets so complex that perhaps algebra would be useful.

11. Let's say that we wished to find out how many 15-amp toasters we could connect to a 75-amp circuit if it already had other loads totalling 30 amps connected to it. We could write 15X, X being the number of toasters, plus 30, equals 75. Here we need to perform two rather simple and obvious operations to both sides. What are they? (Subtract 30, divide by 15.) (Multiply by 15, add 75.)

12. Yes. If we subtract 30 from both sides, we get 15X equals 45. Dividing by 15 gives the solution X = 3 toasters. You might have been able to do the two steps in your head, but how about 3 steps, or four? In what kind of relationships are equations especially useful? (simple) (relatively complex)

13. Right. For example, let's say that in a resistance bridge we found that in connecting five resistors of an equal unknown value in series with another resistor of 2 ohms, we obtained the same total resistance as three of the same value resistors and 8 ohms. We could write 5R + 2 = 3R + 8.

14. Here we have two expressions, both with a term including the unknown resistance, R. To "solve" the equation, we must isolate the unknown on one side, say the left side, and reduce the value of the root to its simplest form, saying in this case that R equals some number. How do we eliminate an R-term from the right side? (Subtract 3R from both sides.) (Divide both sides by 2.)

15. Yes. This would give 2R + 2 = 8. What could be done as the next step to isolate the unknown? (Subtract 8 from both sides.) (Subtract 2 from both sides.)

16. Right, and we would have 2R equals 8 – 2, or 6. By quick inspection, you will realize that R can be completely isolated by dividing by 2 to provide a solution, R = 3 ohms. The past two steps could have been reversed in order, in which case you would have divided both sides by two, operating on each term, then subtracting one from both sides.

17. A formula is a physical law, or useful rule, stated in the form of an equation. The elements of a formula are represented by letters or symbols. In an electrical circuit, for example, you will learn that the voltage across a resistor equals its resistance times the current through it. This rule, Ohm's Law, is written E = I times R, or simply E = I R.

18. If E, the voltage, is the product of I and R, we can "solve" for resistance, R, by dividing both sides by I, the current. After doing so, we have $\frac{E}{I} = R$. Even though R is on the right side of the equation it has been isolated, and just as easily could be written $R = \frac{E}{I}$.

19. Since on different occasions we may know different things about a circuit, and will need to find the unknown quantity, it is useful to be able to solve formulas for any of the quantities which appear in it. "Solving," you may remember, means isolating it on one side of the formula equation. We can say that E = I R, or R = E over I, or I = E over R, and we can arrange the voltage-current power formula in different ways, too.

20. Don't forget that certain operations must be performed before others. For example, operations within parentheses must be done first; you remember, the outside operations must be done on each term inside the parentheses. If you had two series resistors, 10 ohms and a 20-ohms through which 3 amps were flowing, you would write the equation, $E = IR_{1+2}$ as $E = (R_1 + R_2)$; substituting, we get $E = 3(10 + 20)$. In this case, what should we do first? (Add quantities in parentheses $(10 + 20)$.) (Divide by 3 and subtract E.)

21. Yes. The solution to the equation is, of course, 3 amps times 30 ohms, or 90 volts. If the problem had been, say, that there were 3 amps of current through 10 ohms, plus 20 volts drop across another resistor, we'd write $E = 3$ times 10 plus 20. Without the parentheses, however, the multiplication would be performed first, then the addition. What would we get for the overall voltage E, in this case? (20) (30) (50 V)

22. Right. You must treat the quantity in parentheses as a single number. This may not be significant if you are merely adding it to a term. But if you are subtracting it, you must watch the signs of the terms. For example, if you have 12 volts to which you add the voltage quantity $(3 + 5)$ in parentheses, you merely have 12 plus 3 plus 5, and you can drop the parentheses. You may even do this, for example, when adding the quantity $(7 - 4)$ to 12 to get 12 plus 7 minus 4. Look this over.

23. If you wish to subtract the quantity $(7 - 4)$ from 12, however, you must be careful to watch signs. As you have learned, you would write 12 minus the quantity in parentheses $7 - 4$ as $12 - 7 + 4$. Check this over, too.

24. What would you write for the expression 25 minus the quantity in parentheses $(9 - 6)$, if you dropped the parentheses? $[25 - 9 - 6 \ (10)]$ $[25 - 9 + 6 \ (22)]$

25. Right. How would you write 3 times the quantity $(5 - 2)$? $[3(3)$ or $(15 - 6) = 9]$ $[15 - 2 = 13]$

26. Now let's use these procedures in some electronic problems. This series circuit has 3 amperes flowing through two series resistors, one 6 ohms, and one 2 ohms. What is the voltage? Since the total resistance is the sum of the resistances, and the voltage is the product of current times total resistance, we write E volts equals what? $[E = IR; E = I(R_1 + R_2); E = 3(6+2)]$ $[E = IR; E = IR_1 + R_2; E = 3 \times 6 + 2]$

27. What is the total voltage across the two resistors? $(E = 24 \text{ V})$ $(E = 20 \text{ V})$

28. Let's say that two unknown resistors in series, plus a 10-ohm resistance, are found to equal a 22-ohm resistor. We write $2R + 10 = 22$. The formal way to take the first step is to subtract 10 from both sides. This gives $2R = 22 - 10$. A shorthand way to do this, of course, is merely move the term from one side of the equation to the other and change its sign.

29. In transposing the terms involving the unknown for which you are solving to the left

side of the equation and the other terms to the right side, you must remember to be sure that the terms are independent terms, and not part of a polynomial, a collection of terms in parentheses, which is multiplied or divided by some number.

30. And when transposing the term by an assumed addition or subtraction and changing its sign, you must be sure you enter it as an independent term, and not accidentally include it as a term of some binomial which is multiplied or divided by a number.

31. Which of these series of steps provides the correct solution? [$5x - 1 = 3(x + 3)$] ($5x = 3x + 3 - 1$; $5x = 3x + 2$; $5x - 3x = 2$; $2x = 2$; $x = 1$) ($5x = 3(x + 3) + 1$; $5x = 3x + 9 + 1$; $5x = 3x + 10$; $5x - 3x = 10$; $2x = 10$; $x = 5$)

32. Right. In showing division operations in algebra, we generally prefer to use the fraction bar, and to write all of the terms being divided above the bar and the entire divisor as the denominator below the bar. For example, $\frac{3}{5}$ times x times y could be written $\frac{3xy}{5}$. Look this over a few seconds.

33. How could you write $\left(\frac{5}{7}\right)$x times $\left(\frac{2}{3}\right)$y? [$\left(\frac{5}{7}\right)\left(\frac{2}{3}\right)$ xy, $\frac{10xy}{21}$] [$\frac{52}{73}$ xy]

34. How would we write this as an equation? "The reactance X equals the inverse of 2π times the frequency in hertz and times the capacitance in farads." ($2\pi X = fC$) ($X = \frac{1}{2\pi fC}$)

35. Right. If we know the frequency and the desired reactance, how would we solve this formula for the capacitance, C? [$X = \frac{1}{2\pi fC}$] (Multiply by C. $(X_c)(C) = \frac{1}{2\pi f}$; Divide by X_c. $C = \frac{1}{2\pi fX_c}$) (Subtract C. $X - C = \frac{1}{2}$; Add X. $C = \frac{fX}{2\pi}$)

36. Here's another statement we can write as a formula and an equation. "The power used in a resistor equals the square of the current through it, times the resistance." How could you write this? ($W = I^2R$) ($W = \frac{I}{R}2$)

37. Yes. Now, if we had a number of 2-watt resistors, say of 10, 50, and 200 ohms and so on, and we wished to know the maximum current each could pass without exceeding the two-watt rating, we would solve the formula for the current I. What is the first step? ($W = I^2R$; Divide by R; $\frac{W}{R} = I^2$) ($W = I^2R$; Multiply by W; $W^2 = I^2WR$)

38. Yes, and for convention's sake, we will write $I^2 = \frac{W}{R}$. (This side reversal is to put the unknown on the left side for convenience. Formally, it would consist of subtracting one side entirely, and then the other, then multiplying by minus one.) Now we have the square of the unknown by itself. How do we solve the equation completely, that is, obtain the unknown at the first power, isolated? (Divide both sides by 2.) (Take the square root of both sides. $I = \pm\sqrt{\frac{W}{R}}$)

39. Right. For a 10-ohm resistor at 2 watts of power, the current would be the square root of .2 or about .45 amp. The current through a 50-ohm resistor with 2 watts of power would be the square root of $\frac{2}{50}$ or $\frac{4}{100}$, which is .2 amp, or 200 milliamps. What's the maximum rated current for a 200-ohm 2-watt resistor? ($\sqrt{\frac{2}{200}}$ = .1 amp) ($\sqrt{\frac{2}{200}}$ = .316 amp)

Making Equations from Statements

1. In previous programs, you have learned about using equations for solving problems about which statements have been made. The "solution" of each problem is a number which is the value of a previously unknown quantity for a given situation. This value, of course, is expressed in a selected dimension, or unit, such as hours or seconds, dollars, inches or centimeters.

2. In electronics, the units may be watts, amperes, milliamperes, volts, ohms, hertz or cycles per second, millihenries or microfarads. Sometimes the unit is carefully written after the number, and at other times, it is understood and omitted, but it is always an integral part of any answer or solution.

3. In your series of programs on basic electrical circuits, you will learn that the unit of electrical current flow is the ampere—amp, for short. It is often symbolized, as you have seen, by the letter I. The unit of electrical force or pressure is the volt, and it is sometimes written as E or emf. Resistance to flow, written R, is measured in ohms. A current of one amp will flow through one ohm when one volt of emf is applied.

4. What current do you think would flow through one ohm, when connected to 6 volts? (2 amps) (4 amps) (6 amps)

5. Right. You have learned that the relationship, "voltage equals current times resistance," is Ohm's Law. You have already practiced solving for each of the three quantities involved. You have also seen the formula, "watts equal amps times volts." Combinations of Ohm's Law and the watts-power formula which give watts equal I squared times R and watts equal E squared over R are easy to derive. What quantity do you think "watts" measure? (electrical force) (electric power) (electric current flow)

6. Yes, electric power, which is the rate of using or converting electrical energy per unit of time. In your basic electricity programs, you will also learn formulas which state that the inductive reactance, which limits AC current flow and is symbolized X_L, equals 2π times the frequency in hertz, times the inductance in henries. We write this relationship as $X_L = 2\pi fL$.

7. If you wished to find the ohms of reactance, X sub L, for a number of frequencies, F, or inductances, L, you would write their values in place of the letters F and L and perform the indicated multiplications. This is called substituting or plugging in the known values, and finding the unknown value. For example, if the frequency were 100 hertz, which means cycles per second, and the inductance were 10 henries, the reactance would be 2 times 3.14 times 100 times 10, or how many ohms? (6,280 ohms) (62,800 ohms) (628,000 ohms)

8. Let's assume you wished to design a step attenuator which would have a series of reactances

at 100 hertz in steps of 1000 ohms of reactance. We would need to solve the equation $X = 2\pi fL$ for L, to find the unknown inductance values for the selected frequency, and various selected reactance values. How would you solve this formula for L? (Divide both sides by $2\pi f$. $\frac{X_L}{2\pi f} = L$) (Subtract $2\pi f$ from both sides.)

9. Yes. Now if L equals X sub L over 2 pi f, we could say that L equals one over 2 pi f times X sub L, and then find what 1 over 2 pi f equals when f is 100 hertz. A little punching of our calculator or slide rule work gives us one over 628, or 1.59 times 10 to the negative third power, for the value of one over 2 pi f when f is 100 hertz. All we must do now is multiply this figure times 1,000 ohms, then 2,000 ohms, 3,000 ohms and so on, to obtain the inductances needed for our attenuator. What are they? (1.59 hy, 3.18 hy, 4.77 hy, etc.) (1,590 hy, 3,180 hy, 4,770 hy, etc.)

10. Yes. You could set in the 1.59 figure as a constant on your calculator, and very quickly obtain every needed point on a table, or on a graph, since the formula has been solved for the unknown. And you have learned that to solve equations and formulas for a desired quantity, you must isolate it on one side by performing indicated group operations, then adding, multiplying, subtracting, and dividing as required for the isolation.

11. A capacitor allows alternating current to flow through it, and the more capacitance it has, the more current may flow. Conversely, the reactance of a capacitor is greater, as the capacitance is smaller. The formula for such reactance is X sub c equals 1 over 2 pi fC, where f is in hertz and C is in farads, the unit of capacitance.

12. You can see that the reactance X sub c is greater when C is a smaller number of farads. The most commonly used unit of capacitance is microfarads, which are millionths of a farad. If a capacitor had 1,000 microfarads capacity, which is one-thousandth of a farad, how much reactance would it have at 100 hertz? ($X = \frac{1}{(6.28)(100)(1000)}$; X = .00159 ohms) ($X = \frac{1}{(6.28)(100)(.001)}$; X = 1.59 ohms)

13. Right. If we wished to solve the formula X sub c equals 1 over 2 pi fC for C, we would multiply both sides by C, and divide both sides by X sub c to get C equals 1 over 2 pi fX. What capacitance would provide a reactance of 1000 ohms at 60 hertz? ($C = \frac{1}{(6.28)(60)(1000)}$; $C = \frac{1}{377,000}$; C = .00000265 fd or 2.65 mfd) ($C = \frac{1}{(6.28)(60)(1000)}$; $C = \frac{1}{377,000}$; C = 2.65 farad)

14. Yes, about 2½ microfarads produces 1000 ohms at 60 cycles, which means that 5 microfarads is about 500 ohms, 10 microfarads 250, and so on. What would be the reactance of 1 microfarad at 60 hertz? (265Ω) (628Ω) (2650Ω)

15. Remember, micro means one-millionth, milli is one-thousandth, kilo is one thousand, and mega is one million. A picofarad is a U.S. trillionth of a farad, or a millionth of a millionth. Sometimes it's called a micro-microfarad. Microfarads, shown as Mf or Mfd, are often abbreviated with the Greek lower-case letter Mu and English f, and picofarads are sometimes written with two M's or two mu's and f, or just the English letters pf.

16. The equations X sub L equals 2 pi fL and X sub c equals one over 2 pi fC, like the equations E equals IR, P equals EI, P equals I squared R, and so on, are formulas which express approximate or exact relationships between the quantities. It is important, of course, to remember the units intended in each formula, since gross errors can otherwise be made.

17. For example, the AC voltage frequency of a rotating alternator, which is an AC power generator, is determined by its number of pairs of magnetic poles, and its armature speed in revolutions per second. This is written f equals PS. If you were to connect a tachometer to a 60 hertz alternator, and find it is turning at 1800 RPM, how many poles does it have?

18. First, before we make any errors, we must remember that f equals PS means f in hertz, P in pairs of poles and S in revolutions per second, not minutes. Perhaps we should have written it f sub hertz equals P sub pr times S sub RPS, or f equals P sub pr times S sub RPM over 60. We know the frequency and the speed in RPM. What are we asked to find? (the number of seconds per minute) (the number of poles on the alternator) (the number of cycles per minute)

19. Yes. First let's solve for P. To isolate it, we divide by S, and get $\frac{f}{S}$. When speed is RPM, instead of RPS, what would we write? (P equals $\frac{60f}{S_{(RPM)}}$) (P equals $\frac{f}{60S_{(RPM)}}$)

20. Yes. Substituting in P equals 60f over S, the 60 hertz for f and 1800 RPM for S, how many poles are there? [2 pairs of poles (4 poles)] [6 poles (3 pairs)] [60 poles]

21. Yes. How many poles are there in a 60-hertz alternator turning at 1200 RPM? [2 pairs of poles (4 poles)] [3 pairs (6 poles)] [60 poles]

22. In a swimming pool, if wave ripples advanced at 50 inches per second, vibrations at 5 cycles per second would make waves 10 inches apart. Wave motion follows the formula V equals fL or V equals f lambda where V is the wave velocity, f is the frequency and L or lambda, the Greek letter for L, is wave length. All of the quantities, of course, must be in the same set of units.

23. If water waves moved at 50 inches per second, what frequencies would be used to make waves that are 2, 4, 6, and 8 inches long, between crests? First, we solve the equation for the unknown frequency, then plug in the given numbers. How do we write V equals fL when solved for f? (f equals VL) (f equals $\frac{V}{L}$)

24. Right. f equals V over L or f equals V over lambda. On our slide rule or calculator we set in the constant for V, which is 50, then in turn divide it by the various given values of wavelength. What are the frequencies required for 2, 4, 6, and 8 inch waves? (25 hz, 12.5 hz, 8.33 hz, 6.25 hz) (20 hz, 40 hz, 60 hz, 80 hz)

25. Yes. Dividing inches per second velocity by inches per cycle, or wave, we get cycles per second, or waves per second. In electronics, we seldom need to work with water waves, but

often must deal with radio waves. Radio waves, like light waves, travel at 186,000 miles per second, or about 300,000 kilometers per second. The same formula is used, V equals f times lambda, so that the product of the frequency and wave length is always this constant velocity.

26. Since the velocity of the radio waves is 300,000 kilometers per second or is 300 million meters per second, what is the wave length of one megacycle radio frequency? We can see at once it is 300 meters. But if we wish to derive a table or graph of wave lengths, we need to use a little simple algebra. What is V equals f times lambda solved for lambda? (λ equals fV) (λ equals $\frac{V}{f}$) (λ equals $\frac{f}{V}$)

27. Yes, the wavelength lambda or L is $\frac{V}{f}$ and if we use meters, megacycles per second, or megahertz, we can substitute 300 for V to get lambda equals $\frac{300}{f}$ in megahertz. The wave length of 1 megacycle per second is 300 meters, as we said; 100 megahertz has a 3-meter wave length, and 300 megahertz has a 1-meter wavelength.

28. If you wish to design a TV antenna measured in feet, you can use 984 divided by frequency to get wave length, since radio waves travel 984 million feet per second. A 100 megahertz antenna would be 9.84 feet long for full-wave, or half as long, about 4.9 feet, for a half-wave element.

29. Most electronic formulas and equations are relatively simple, with only a few variables in simple relationships. The techniques of algebra can also deal with complex relationships of several variables. Sometimes such problems occur in electronics. For example, if you had a radio shack equipped with a 300-watt output transmitter which operated at 40% efficiency, and you had a 4225 BTU per hour electric coil heater and 600 watts of lights, all on one 120 volt circuit, what size fuse would you need?

30. First, we would have to write an equation. What group of quantities equals what quantity? Let's say that the total power available through the fuse equals the sum of the three electrical loads. How do we express the power available? ($\frac{120 \text{ volts}}{4225}$) [(120 volts)(40%)] [(120 volts)(fuse current rating)]

31. Right, the power available is voltage times current available, and this is determined by the fuse rating, which is the unknown value in the problem. The three electrical loads on the circuit include a 40% efficient transmitter. The output is 300 watts, and this is 40% of the power input, which then must be 300 divided by .40.

32. The electric heater is rated at 4225 BTU per hour. Since one watt of electrical power is converted in resistance heaters to 3.2 BTU per hour, the power input to the heater is 4225 BTU, divided by 3.2 BTU per watt. Can we now set up the equation? (Yes) (Never)

33. We now say that the available power, 120 volts, times the fuse rating, equals 300 watts radio power divided by .40, plus 4225 BTU's of heat, divided by 3.2, plus 600 watts for lights. What is the total load in watts? [(120).40 + $\frac{425}{3.2}$ + 600; 1760 watts] [$\frac{300}{.40}$ + $\frac{4225}{3.2}$ + 600 = 750 + 1320 + 600 = 2670 watts)

Solving Linear Equations

1. In earlier programs you solved problems by using formulas that contained letters. The formula showed how the quantities represented by the letters were related. You could replace all of the letters with values except for one, say x or y, or whatever. This was your unknown which you were solving for. Even though you will always solve for one unknown at a time, it is not always best or easiest to set up just one equation with one unknown.

2. For example, suppose you know that the total resistance of two resistors, call them resistor x and resistor y, is 24 ohms. The first equation, or relationship, you make is that $R_x + R_y$ = 24 ohms, but you still know nothing about their individual values. But if we had also said that R_x is equal to 5 times R_y, the solution comes quickly. What does R_x equal? (10 ohms) (4.8 ohms) (20 ohms)

3. Correct, and R_y equals 4 ohms. We have solved a problem involving two unknowns and two equations. We solved it by a method called substitution. We took the value for one unknown as it was given in one equation and substituted it in place of the same unknown in the other equation.

4. Our two equations, or relationships, were that $R_x + R_y$ = 24 ohms and that $R_x = 5R_y$. By substituting $5R_y$ in the first equation for R_x, we got $5R_y + R_y$ = 24 ohms, or R_y = 4 ohms and R_x = 20 ohms. This process allowed us to simplify the problem into one having just one unknown quantity. What do we call this process? (solution) (correlation) (substitution)

5. Yes. Besides substitution, we can solve a problem with two unknowns and two equations by using graphical methods and by using the algebraic difference between the two equations to eliminate one unknown. This last procedure is called solving simultaneous equations.

6. Let's consider how we can use graphical methods. Here is a graph of $x = 2y$. By looking at it we can predict what the value of y will be for any given value of x. When $x = 0$, what will be the value of y? (0) (1) (2)

7. Correct, because even if x is always twice as large as y, as the equation shows, when x is 0, y is also 0. A graph is composed of the intersection of a horizontal line and a vertical line. We call these lines the x and y axes, respectively. Their intersection is called the origin and is 0 in value for both. The x axis is positive to the right of the origin and negative to the left, while above 0 on the y axis is positive and below 0 is negative. If x is increased, which way will it move on the axis? (upward) (downward) (to the right)

8. Yes, and as the value of y increases, it will move upward. Because x and y are letters which stand for values which can vary, they are called variables. Letters or numbers, on the other hand, which represent known values are called constants.

9. In the graph for the equation $x = 2y$, we said that we could predict the value of y whenever we change the value of x. The value of y is dependent then on the value of x. A variable whose quantity varies according to the value of another variable is called a dependent variable. What do you think we would call x, the variable whose value in this case determines what y is? (arbitrary unknown) (independent variable)

10. Right. We usually choose to call the independent variable x, and its value on a graph will change horizontally. The dependent variable, usually called y, will then vary graphically up and down. Consequently, we generally think in terms of how much vertical change occurs with a change in the horizontal value.

11. Another term is function. We say that the dependent variable is a "function" of the independent variable. Let's say you use a gallon of gas for each 20 miles you drive. A longer trip would use more gas. We can show this as 1 mile equals 20 gallons, or that one gallon equals $\frac{1}{20}$ the number of miles. What variable would be considered a function of M, or miles driven? (x) $(\frac{1}{20})$ (G)

12. Yes. A graph of our example of the resistors would be a graph of the equation R_x plus $R_y = 24$. If resistor x has a higher resistance, then resistor y obviously would have less. Also, if either is 0, then the other is 24 ohms. As you can see, the graph of the equation is a straight line, like the graph we saw of $x = 2y$, but not in the same direction. What might we call equations which graph as a straight line? (curvy) (linear) (circular)

13. Correct. You can usually identify a linear equation (one which graphs as a straight line) because it has only "first order" terms for its unknowns. This means that each term is to the first power, with the exponent "1" unwritten, but understood. Examples of these would be $6x$, $22y$, $\frac{1}{3}$ b, and so on. Terms like x^2, πr^2, and c^3 are not "first order."

14. Here are some graphs of linear equations. What do you notice about them? (They are all curved.) (They slope down to the right.) (They're straight, slope up to the right.)

15. Right. Other graphs, like the ones shown here, can slope down to the right. This happens because when the equation is solved for y, x is negative. A downward slope is called a negative slope.

16. You'll remember that the dependent variable is usually called y, and changes position on a graph vertically. If you solved the equation $3x = 4y + 6$ for y, what would you have? $[y = 4(3x - 6)]$ $[y = \frac{4}{6 - 3x}]$ $[y = \frac{3x - 6}{4}]$

17. Correct. Let's look now at how a line can represent an equation. Take the equation $x = y$. On a graph, as the value of x increases by moving to the right, the value of y will also increase and move upward. If you begin at the center of the graph with $x = 0$ and $y = 0$, you will then move on an upward slope through $x = 1$ and $y = 1$ to $x = 2$, $y = 2$, and so on.

18. In the example, $x = 2y$, when $x = 0$, y obviously equals 0; if $x = 2$, then $y = 1$; if x is 4, $y = 2$. What would y be if $x = 6$; and would it be graphed on a straight line through the other points?

(x = 6; y = 3; Yes) (x = 6; y = 4.735; No)

19. Yes. An equation like x = y + 2 is a little different. After graphing points like x = 2, y = 0; x = 3, y = 1; x = 4, y = 2; and x = 5 and y = 3, we can see that even though it is a straight line, it does not go through the origin.

20. You might see that we refer to a point in alphabetical order: first x, then y. A point p having x = 3 and y = 4 would be shown as 3 comma 4 in parentheses. If a point was said to be a parentheses 6 comma 3, what are x and y? (x = 6, y = 3) (x = 3, y = 6)

21. Right. The numbers that refer to a point on a graph are called its coordinates. The coordinates of the point where x = 5 and y = 3 are parentheses 5 comma 3. The vertical coordinate or y value is called the ordinate. The x value or horizontal coordinate is called the abcissa.

22. Before we go on, let's review a little. An equation with only "first order" terms is called what? (hyperbola) (linear) (second rate)

23. Yes. We usually graph which variable on the y axis? (independent) (dependent)

24. What are the correct order and names of the coordinates of a point on a graph? [(x,y) ordinate, abcissa] [(x,y) abcissa, ordinate]

25. Study this graph carefully and decide which equation is shown. (2x = y + 3) (x = 2y + 3) (2x = 3 – y)

26. Right. When working with a problem involving two unknown quantities, we need two relationships or equations to find any specific solution. You can graph these two equations and see if they cross each other. If they do cross, the point of intersection has x and y values which are true for both equations and provides a solution which is simultaneously true for both. Although not always the quickest method, graphing is one good way to find a solution.

27. Here is a graph for the equations x = y and x + y = 6. What are the coordinates of the intersection, or the solution? [(3,3)] [(3,6)] [(2,4)]

28. Correct, although you could have guessed that if x equalled y and their sum was 6, they were both equal to 3. What about the solution of x = 2y and x + 2y = 8? [2,4)] [(3,3)] [(4,2)]

29. Right. So far, we've not gone through the steps required for plotting equations, but you can see that only two widely separated points are needed to plot a straight line. Unless x is just some multiple or fraction of y, one easy method of selecting two such points on the graph is to let x=0 and evaluate y, and then let y=0 and evaluate x.

30. A little thought will show that graphic solutions do not always provide the quickest or most expedient means of finding the solution to two equations in two unknowns. In fact, the greatest use of plotting the intersection of the two equations is often in the visual meaning given to the abstract algebraic relationships involved. For example, what is the solution shown here in the graph of 2x = 3y and 2x + y = 8? [(0,0)] [(3,2)] [(2,4)]

31. Yes. Another method you can use is the subtraction method. This gives an algebraic solution which eliminates one of the unknown quantities involved. This serves the same purpose as the substitution method described earlier, but is more generally useful.

32. If you had a pair of equations, $x = y$ and $x + y = 6$, you could change the first equation to $x - y = 0$ by subtracting y from both sides. Now write one equation above the other. Remember that you can add equals to equals, or subtract equals from equals, and the result will still be an equality. Adding $x - y = 0$ and $x + y = 6$ would eliminate the y term. Subtracting the two would eliminate the x term. First, let's try addition. What do we get? $(2x = 6; x = 3)$ $(2x + y = 6; x = 2)$ $(x = 6 + 0; x = 6)$

33. Yes, and after finding the value of one unknown, you can substitute it in either equation to find the value of the other unknown. How would you combine $2x + y = 9$ and $x + y = 6$ to solve for x? (Subtract; get $x = 3$) (Add; get $3x + 2y = 12; x = 4$)

34. Yes. But what if we have these two equations, $3x = 2y + 7$ and $2x + y = 0$. First, let's get all of the unknowns on the left side of the equations and the plain number terms on the right. (Later, you will find the so-called "standard" form of some equations calls for a zero on the right side.) In this case, what do we do to place all unknowns on the left and all the constants on the right? (Add $2x$ to equation II; add $4x + 4 = 0$) (Subtract $2y$ from equation I; get $3x - 2y = 7$.)

35. Right, but we still don't have two equations we can add or subtract to eliminate one unknown. We can, however, multiply both sides of one, or both, equations so that the coefficient of one of the unknowns is the same in both equations. If we multiplied both sides of the second equation by 2, which unknown would this help eliminate? (x) (y) (z)

36. Yes. We now have $3x - 2y = 7$ and $4x + 2y = 0$. What next? (Add equations; $7x = 7; x = 1$) (Subtract equations; $x = 7$)

37. A little more difficult one is $6x + 3y = 4$ and $10x - 6y = 3$. What now? (Multiply equation I by 2, add equations.) (Add algebraically, get $16x = 16; x = 1$)

38. Yes. Now $12x + 6y = 8$ and $10x - 6y = 3$. Added this gives us $22x = 11$. x will then equal what? (2) (1) (½)

39. Right. We have solved two equations with two unknowns by three different methods: 1) by substitution; that is, by substituting one unknown as a function, or in terms of the other, when it's simple to do so; 2) by graphing, which provides a visible, understandable picture, and shows the solution at the intersection or crossing point of the equation lines; and 3) by subtraction, or algebraic addition, to provide an algebraic solution.

40. When we have two unknowns, and two linear equations which can solve them, what are they called? (coexisting equations) (simultaneous equations) (congruative equations)

Solving Simultaneous Equations

1. In the previous program, you learned of several methods for dealing with two linear equations and two unknowns. These kinds of equations, you recall, were termed "simultaneous equations." One of the most generally used methods for solving them is subtraction, or algebraic addition, in order to give an algebraic solution. In this program, we will deal further with simultaneous equations and this method of solving them.

2. The purpose of this method is to eliminate one of the variables. For example, $x + y = 6$ and $x - y = 2$ could be solved by adding them together. Which variable would be eliminated? (x) (2) (y)

3. Yes. We can multiply either equation or both so that the coefficient of one of the unknowns will be equal. For example, how would you go about solving the equations $4x + 9y = 8$ and $2x + 6y = 4$? (Add them together.) (Divide both by 2 and subtract.) (Multiply the first equation by 2, then subtract.)

4. Right. Suppose now that you are buying electrical supplies and you know that fifty resistors and fifty capacitors together cost $10.50. At the same time, you know that 80 capacitors and 30 resistors cost $12.30. How would you go about determining the cost of each capacitor and resistor?

5. First we must set up our relationships, or equations. We can say that 30 resistors, call them R, plus 80 capacitors, or C, equal $12.30 and that 50 C plus 50 R = $10.50. After writing them down, you can see that both of the equations will have to be multiplied in order to eliminate one of the unknowns. Suppose you want to eliminate the resistors and find the cost of a capacitor. What would you multiply each equation by? (Multiply the first equation by 5 and the second equation by 3.) (Multiply the first equation by 5 and the second equation by 8.)

6. Correct. If we multiply the first equation by 5 and the second equation by 3 and subtract, what is the result? $(250\ C = \$30.00; C = \$.12)$ $(550\ C = \$93.00; C = \$.106$ or $\$.11)$

7. Right. Here's a little harder problem. We know that in a circuit, the current I is equal to the voltage, called E, divided by the resistance R. We also know that the total power in a circuit, or W, equals the current, in amps, squared times the resistance R. How would you write these relationships? $(I = \frac{R}{E} \ ; W = \frac{R^2}{I})$ $(\ I = \frac{E}{R} \ ; W = I^2 R)$

8. Yes. Now suppose we know that W = 2.7 kilowatts, or 2700 watts, and that the voltage E = 180 volts. How would we begin to solve for the current and resistance in the circuit? We must eliminate one of the unknowns, and the easiest is I. If $I = \frac{E}{R}$, then we might say

that $W = E^2/R^2$ times R. Is this true? (Yes) (No)

9. Correct. If we do this, we now have only one equation in one unknown. In this case, the unknown is R and we can now find its value. We saw earlier that dividing a number raised to a power by that number gives the number to a power one less than the original. So if we have E^2/R^2 times R, and cancel the R's, we get E^2/R. This makes our equation read $W = E^2/R$.

10. If we multiply each side by R and divide each side by W, we arrive at $R = E^2/W$. All we have to do now is replace E and W with their given values and solve. What is the result? ($R = (180)^2/2700 = \frac{36,400}{2700}$; $R = 15$ ohms) ($R = (2700)^2/180 = \frac{7,290,000}{180}$; $R = 40,500$ ohms)

11. Yes. And if we replace the variable R in $I = \frac{E}{R}$, with 15 ohms, we find the current is 12 amperes. The problem you have just worked is one involving common relationships that occur frequently in electronics.

12. Let's try another problem. We know that the voltage E equals the current I times the resistance R, or $E = IR$. This is known as Ohm's Law. Another common formula in electronics is that the power W equals the voltage E times the current I, or $W = EI$. If you substituted the first equation into the second equation, what equation would you have that we just worked with? ($W = I^2R$) ($R = W^2/E$) ($E = RIW$)

13. Yes. Now assume that you have two circuits, each with the same resistance. There is a voltage E across circuit 1 of 18 volts and it draws a current of 3 amps. Circuit 1 has 2 resistors, say R_1 and R_2, where R_1 has only 20% the resistance of R_2. How would you express this information about circuit 1 in two equations? ($R_1 = .20 R_2$; 18 volts = 3 amps $(R_1 + R_2)$) ($R_1 = .2 R_2$; 18 volts = $(R_1 + R_2)/3$ amps)

14. Correct. We also know that there is 9 volts across the second circuit and that it has only one resistor but with the same total resistance of circuit 1. What formula would show how we would find the current in circuit 2? ($I = \frac{R}{E}$) ($W = I^2R$) ($I = \frac{E}{R}$)

15. Yes. Before we can find the current in circuit 2, though, we must determine what the resistance is in circuit 1. If $E = IR$, how do we find this resistance, and what is the result? ($R = \frac{E}{I} = \frac{18 \text{ volts}}{3 \text{ amps}} = 6$ ohms) ($R = \frac{I}{E} = \frac{3 \text{ amps}}{18 \text{ volts}} = 1.66$ ohms)

16. Before finding the current, let's see what the values of R_1 and R_2 are. We now know that $R_1 + R_2 = 6$ ohms and $R_2 = .2 R$. What do R_1 and R_2 equal? ($.2 R + R_2 = 6$ ohms; $1.2 R_2 = 6$ ohms; $R_2 = 5$ ohms, $R_1 = 1$ ohm) (6 ohms = $.8 R_2$; $R_2 = .75$ ohms; $R_1 = 5.25$ ohms)

17. Correct, and if there are 9 volts across circuit two and 6 ohms resistance, what is the current in circuit two? (2.3 amps) (1 amp) (1.5 amps)

18. Correct. You can see that once you are able to establish the relationships, or equations, between the unknown quantities, the problems become fairly easy to solve. What do we call this

method of working with two equations to eliminate an unknown? (solving by simultaneous equations) (find the mean) (graphing the coordinate)

19. Right. One way to measure wind velocity is to measure the speed of sound along a line between two microphones, with a source of sound between them. Dr. Goldstein put two small radio transmitters with microphones 100 meters apart in a line with the wind direction, and connected one earphone to each of the receivers.

20. Then he walked between the microphone with a clapstick until he heard the sound coming from both receivers at the same instant. If the speed of sound was 300 meters per second, and Dr. Goldstein was 48 meters from the nearest microphone, what was the wind velocity?

21. Net sound velocity equals the distance travelled divided by time in meters per second. Therefore, it would be 300 meters per second plus or minus wind velocity W. The time T is the same for both distances, 48 and 52 meters, but the net sound velocity for the 48-meter distance was different. What was it? (W) $(300 + W)$ $(300 - W)$

22. Right. It was slower because it travelled a shorter distance. We could say, then, that this velocity equalled 48 meters divided by the time required, T, whatever it was. So $300 - W = \frac{48}{T}$. What else do we know? $(T = 300 + W(48))$ (nothing) $(300 + W = \frac{52}{T})$

23. Right again. We now have two equations. $300 - W = \frac{48}{T}$ and $300 + W = \frac{52}{T}$. If we were to add these two equations, would we eliminate the unknown "W"? (Yes) (No)

24. Yes. We get $600 = \frac{48}{T} + \frac{52}{T}$. What would you suggest now? (Multiply both sides by T.) (Subtract 600 from both sides.) (Cross multiply $(\frac{48}{T})(\frac{52}{T})$.)

25. Yes. And we'd get $600\,T = 48 + 52 = 100$, and $T = \frac{1}{6}$ second. How do we find W now? (Substitute $\frac{1}{6}$ for T in $300 + W = \frac{52}{T}$.) (Multiply T times $300 + W$, then add 52.)

26. Correct. This would give $300 + W = \frac{52}{1/6} = 6$ times $52 = 312$. What is the wind velocity? (48 meters/sec.) (52 meters/sec.) (12 meters/sec. or 27 MPH)

27. Correct. Let's try another example. The total capacitance, in farads, of a number of capacitors in parallel is equal to the sum of the individual capacitances. You know the following: The total capacitance of 3 capacitors, say C_1, C_2, C_3 is 48 microfarads. The capacitance of C_1 is 20% that of C_2, and the capacitance of C_2 equals ½ that of C_3. What are the equations which express these relationships. $(C_1 + C_2 + C_3 = 48$ microfarads; $C_1 = .2\,C_2$; $C_2 = .5\,C_3)$ $(C_1 + C_2 + C_3 = 48$ microfarads; $C_2 = .2\,C_1$; $C_1 = 5\,C_3)$

28. Right. Now you need to eliminate one of the unknown capacitances. Which would probably be the easiest? (C_1) (C_2) (C_3)

29. Yes. Since C_1 is 20% or $\frac{1}{5}$ the value of C_2, and C_2 is ½ the value of C_3, C_1 is also equal to $\frac{1}{10}$ the capacitance of C_3. Which equation would show these relationships? $(.1\,C_3 + .5\,C_2 + C_1 = 48$ microfarads) $(.1\,C_3 + .5\,C_3 + C_3 = 48$ microfarads)

30. Correct. What is the value then of C_3? (30 microfarads) (3×10^{-5} farads) (either is correct)

31. We can then replace the value of C_3 with 30 microfarads and find that C_1 = 3 microfarads and C_2 = 15 microfarads. Now let's perform a few simple problems for review. How would you find the two numbers, say x and y, whose sum is 29 and difference is 8? (Add, get $2x = 42$; $x = 21$, $y = 8$) (either) (Subtract, get $2y = 16$, $x = 21$)

32. Yes. One resistor has 30% the resistance of another and their total is 19.5 ohms. What are the two resistances? (15 ohms, 4.5 ohms) (13 ohms, 6.5 ohms) (10 ohms, 5.5 ohms)

33. Correct. If W equals I squared R and E equals IR, express the power W in terms of voltage and current. ($W = E^2/I^2$) ($W = IR$) ($W = EI$)

34. You are given the problem of finding the solution to $22x - 13y = 8$ and $11x + 8y = 36$. Which method would probably be faster? (using simultaneous equations to solve for one unknown) (graphing)

35. Correct. An important feature of using simultaneous equations is to be able to express the information given in algebraic equations. How would you show that an unknown quantity when multiplied by 6 is equal to 4 times another unknown divided by the square root of two? ($\sqrt{2x} = \frac{(6)(4)}{y}$) ($6x = 4y/\sqrt{2}$)

36. Yes. Which of the following is known as Ohm's Law? ($P = EI$) ($I = \left(\frac{Q}{C}\right)R$) ($E = IR$)

37. Yes. You are given this information: an airplane costs 5 times as much as a certain automobile. Two of the airplanes cost $10,000 more than 6 of the cars. What are the equations which express this information? ($A = 5$ C; 2 A $= 6$ C $+ \$10,000$) ($5$ A $= $ C; $\frac{A}{2} - \$10,000 = 6$ C)

38. Correct. If you solve these two equations for the price of the car, what is the result? (C = $2,500; A = $12,500) (A = $10,000; C = $5,000)

39. Yes. You have been working so far with "first power" or linear equations. In the next program you will learn about solving quadratic equations which deal with squared values and which can have more than one possible answer.

40. This is the end of your program on solving simultaneous equations. You will probably need to repeat it several times to make sure you understand and remember the uses of these equations.

Solving Quadratic Equations

1. In a previous program, you learned about numbers multiplied times themselves. Numbers which are squared, cubed, or raised to the fourth or fifth power, for example, are used to solve real problems today, and they occur in algebraic relationships. What do we call an algebraic statement of the relationships between quantities? (an equivocation) (an equation) (an equestrian)

2. Good. You will recall that linear equations contain only first-order terms, and are called "linear" or "first-order" equations. An equation which contains terms up to the second degree, that is, "squared," is called a "quadratic" equation. The reason mathematicians say "quadratic" for only second degree equations is based on area matters, usually involving "quadrangles" or rectangles.

3. "Quad" means "four," "quadrant" means fourth; but the highest terms in a quadratic equation are what? (first degree) (second degree) (fourth degree)

4. Correct. The standard form of a quadratic equation is with the terms arranged in descending order on the left side of the equation and with zero on the right side, as shown here. Which is the standard form of $2x^2 = 4x - 6$? $(2x^2 - 4x + 6 = 0)$ $(4x + 6$ equals $2x^2)$

5. What is the standard form of $x^2 = 16$? $(x^2 + 16x + 16$ equals $0)$ $(x^2 - 16 = 0)$

6. Yes. Sometimes a quadratic equation doesn't have a first order term or a constant term, so we don't write them. What's the standard form of $10x^2 = 7x$? $(10x^2 - 7x = 0)$ $(x^2 - x + 3 = 0)$

7. Yes, but in this case the x could be factored out, and one root would be x = 0. The remaining root, for practical purposes, might be considered a linear, or first-order, equation.

8. Let's call the general expression of the standard form of a quadratic equation, $AX^2 + BX + C = 0$. A, B, and C can be any given numbers, as shown here. If you had the equation $6 - 3x = -2x^2$, which coefficient value is the B in the standard form? (6) (- 3) (2)

9. Yes. When the B coefficient of the first-order term is 0, such as in $x^2 - 16 = 0$, the equation is easy to solve. The formula becomes $AX^2 + C = 0$, and $X^2 = \frac{-C}{A}$ after subtracting C from both sides and dividing both sides by A. Then what does X equal? $(X = \sqrt{\frac{C}{A}})$ $(X = \pm\sqrt{\frac{-C}{A}})$

10. Right. Of course, if either C or A is a negative number, you will get an imaginary root. In all of our problems, we will deal with real numbers.

11. In $x^2 - 9 = 0$, the left side can be factored into $(x + 3)(x - 3)$, giving us x equal to   - 3.

and + 3. We could also solve for x by adding 9 to both sides, to get $x^2 = 9$. What's $\sqrt{9}$? (3) (4½) (± 3)

12. Right. And if we had the equation $4x^2 - 25 = 0$, we'd solve for x^2, getting $x^2 = \frac{25}{4}$. Take the square root of both sides. What would we find? $(\frac{25}{4})$ (6¼) $(\pm \frac{5}{2})$

13. Right. Most of the time, however, we'll have all three kinds of terms in our quadratic equation. In this general case, we should look quickly at the coefficients to see if they will permit simple factoring. For example, with two binomials ax + m and dx + n multiplied together, we will get adx^2 + (an + dm)(x + mn). Do you think you could factor $12x^2 + 27x + 15$? (looks likely) (seems impossible)

14. Yes. a and d could be 3 and 4; and m and n could 3 and 5. In fact, $12x^2 + 27x + 15$ can be factored into (3x + 3) and (4x + 5). The sum of the cross-products does give 27x for the middle term. There is a formula which you can memorize easily and which solves second-order equations routinely. What would you guess this formula is called? (binomial formula) (triatic formula) (quadratic formula)

15. Right. First, a few frames of factoring practice. What are the roots of $x^2 + 8x + 12 = 0$? (x = - 8; x = - 12) (x = - 2; x = - 6) (x = 2; x = 6)

16. Yes. The binomial factors are x + 2 and x + 6. This product is equal to 0, so either x + 2 = 0, or x + 6 = 0. If x + 2 = 0, we subtract 2 from both sides, and x = - 2.

17. What are the roots of $x^2 - 15x + 54 = 0$? (x = 3; x = 5) (x = 5; x = 9) (x = 6; x = 9)

18. Yes. The trinomial expression factors into (x - 6)(x - 9), and it equals 0, giving roots of 6 and 9.

19. The standard form of a quadratic equation is $AX^2 + BX + C = 0$. To get the roots of X, you divide $(- B \pm \sqrt{B^2 - 4AC})$ by 2A. Perhaps you can set it to some jingle, as a mnemonic trick, like: - B plusorminus squarerootofB^2 minus 4AC allover 2A.

20. The quadratic formula tells us that the roots of X in the equation $AX^2 + BX + C = 0$ are . . . -Bplusorminus — squarerootofB^2 — minus4AC — allover what? (AC) (2A) (2B)

21. Yes. Now let's try out the formula on $x^2 - 5x + 6$. A is 1, B is - 5, and C is 6, so x = - (- 5) $(\pm \sqrt{25 - 4(1)(6)})$ all over 2. When we do the indicated multiplications and subtractions, we get what? $(x = \frac{+5 \pm \sqrt{1}}{2})$ $(x = \frac{-5 \pm \sqrt{4}}{2})$

22. Yes. Since $\sqrt{1}$ is ± 1, we have both $\frac{5 + 1}{2}$ and $\frac{5 - 1}{2}$ as roots. What are these two quantities? (x = 3; x = 2) (x = 4; x = 5)

23. Yes. Now try $2x^2 - x = 1$. This is not in standard form; it should be $2x^2 - x - 1 = 0$. Then what do we have after a substitution into the formula? $(x = \frac{1 \pm \sqrt{1 - 4(2)9 - 1)}}{2(2)})$ $(x = \frac{-1 \pm \sqrt{1 - 4(2)(1)}}{2(2)})$

24. Yes, and evaluating this expression, we get $\frac{1 \pm \sqrt{9}}{4}$ which is $\frac{1 \pm 3}{4}$. What are the roots? ($x = -\frac{1}{2}$; $x = 1$) ($x = -1$; $x = -2$)

25. Right. As you can see, all you need to do is write the equation in standard form, substitute for A, B, and C in the quadratic formula, and do the indicated operations. In general, if the quantity under the radical symbol (called the discriminant), is a perfect square, like 9, 16, or 25, you should go ahead and do all the arithmetic operations. If it's some irrational number, you may usually show your solution by just leaving the radical in the answer.

26. Since you know the multiplication table, you already know that 1, 4, 9, 16, 25, 36, 49, 64, 81, and 100 are perfect squares. It may help to memorize that 121, 144, 169, 196, 225, and 256 are perfect squares for 11 through 16, and 625 is the square of 25. Naturally you recognize that 100, 400, 900, 1600, 2500 and so on are the squares of 10, 20, 30, 40, and 50. After studying these numbers, guess what the square root of 484 is. (18) (22) (28)

27. Yes. You may have access to a small electronic calculator, so you'd think that it would hardly be necessary to do all this drill and memory work. There are several reasons: some test examiners may not permit calculators and prefer that you use tables, mental work can be more satisfying, and if you make a mistake in operating your calculator, a rough mental check may reveal it.

28. If we had a quadratic equation given as $10x - x^2 = 25$, how would we use the formula? First, the standard form: arranging terms in proper order, we get $-x^2 + 10x - 25 = 0$, but for convenience, let's multiply through by -1 and get $x^2 - 10x + 25 = 0$.

29. The values of A, B, and C are 1, -10, and 25. After working the formula, we see immediately that the discriminant is 0. Therefore, what are the roots? ($x = 5$; $x = 5$) ($x = 5$; $x = 10$) ($x = 10$; $x = 10$)

30. Yes. Here's a quadratic problem: Bob and Bill each agreed to mow half of the lawn, which is 90 by 120 feet. Bob mowed around the outside of the lawn, mowing a wider and wider strip. How wide was it when he stopped to let Bill mow the rest?

31. The inside rectangle is half as large in area as the entire rectangle which is 90 by 120 feet. Now we can call the mown strip x feet wide. Then, the area of the inside rectangle is the product of its remaining length and width; or $(90 - 2x)(120 - 2x)$. This remaining area is also half of 90 times 120, or $\frac{(90)(120)}{2}$. Now we have an equation. What is it? ($\frac{(90)(120)}{2} - 4x^2 = 90 + 120$) [$(90 - 2x)(120 - 2x) = \frac{(90)(120)}{2}$]

32. Right. Now let's perform the indicated operations. Multiplying the binomials, collecting terms, and putting them into order, we get $4x^2 - 420x + 10,800 = 5400$. That's still not standard form. We need a 0 on the right side, so we subtract 5400 from both sides and get what? ($4x^2 - 420x + 5400 = 0$) ($4x^2 + 120 - 5400 = 0$)

33. Yes. And just for further simplification, let's divide everything by 4, and get $x^2 - 105x + 1350 = 0$. Which is the quadratic formula applied to this equation? ($x = \frac{105 \pm \sqrt{105^2 - 4(1350)}}{2}$) ($x = \frac{1350 \pm \sqrt{1350^2 - 4(1)(105)}}{2(105)}$)

34. Correct, and this is $x = \dfrac{105 \pm \sqrt{11,025 - 5400}}{2}$. After subtracting, we find the discriminant is the square root of 5,625. What do you think this equals? (45) (55) (75)

35. Right. 105 plus or minus 75, divided by 2, is either 90 or 15. Obviously, Bob isn't going to mow a strip 90 feet wide around a yard that is 90 by 120 feet, so he stops when he makes enough circuits to mow a 15-foot belt around the yard.

36. This would leave a 60 by 90 foot remainder of uncut grass in the center. Who mows the remainder? (Bob) (Bill) (nobody)

37. A bomber airplane flew at a constant airspeed to a target 1800 miles away, against a jetstream headwind of 100 miles per hour, but flew back with it as a tailwind. The round trip took 6 hours. What was its airspeed? It's not exactly 3600 miles divided by 6 hours, or 600 miles per hour. Rather, its somewhat over this speed, because the headwind trip took longer than the tailwind return. The wind caused a slight net loss of time. Let's do the algebra. If the airspeed is x, the outbound trip took 1800 over x - 100 hours and the return trip 1800 over x plus 100 hours, both trips totaling 6 hours. The equation then, should be what? $(\dfrac{1800}{x - 100} + \dfrac{1800}{x + 1800} = 6$ hours) $(\dfrac{1800 + 1800}{x + 100 - 100} = 6$ hours)

38. Yes, but this is hardly in standard form. Multiplying both sides by $(x - 100)(x + 100)$ we would get $1800 (x + 100) + 1800(x - 100) = 6(x + 100)(x - 100)$. Then we need to perform the indicated multiplications and additions. We can then transfer by subtraction to get the standard form. Would you say that $1800x + 1800x = 6(x^2 - 10,000)$ is a correct intermediate step? (Yes) (No)

39. Yes. Now to simplify a little. Let's divide everything by 6, add the x terms, and get $600x = x^2 - 10,000$. With a little subtraction, and multiplying by - 1, we get $x^2 - 600x - 10,000 = 0$ in the standard form. Applying the quadratic formula, you can see this will be over 616 miles an hour.

40. Don't forget, you should repeat this lesson several times to be sure you understand and remember all of the rules of algebra you have learned.

Trigonometry in Electronics

1. You've heard of the mathematical field of trigonometry. From its name, you can tell perhaps that it deals with angles and triangles. A triangle, you know, is an enclosed geometric figure with 3 straight sides. It has 3 other features, too. What are they? (three angles) (three diagonals) (three bases)

2. Yes, of course. And these angles may be of several kinds: acute angles, right angles and obtuse angles. Which of these is an acute angle? (∠) (∟) (◝)

3. Yes. In electricity and electronics we often deal with alternating current in "reactive" circuits. Inductors or capacitors in reactive circuits cause the current to get "out of step," or out of exact synchronism, with the voltage. This is called being "out of phase." We say there is a "phase angle" between the voltage and current, and to work with this phase angle, we need to know something about trigonometry. Trigonometry is partly based on right triangles. A right triangle has one right angle. Which of these is a right angle? (∠) (∟) (◝)

4. Yes. In this program you will review some of the things you learned in algebra, and most of it is similar to the material in a response-type program in an algebra series, so it may be quite familiar. To check your recollection, can there be two right angles in a triangle? (Yes) (No)

5. Correct. A right angle is formed by the intersection of two perpendicular lines which form four equal angles. Mathematicians have chosen to say that a right angle contains 90 degrees. An acute angle contains less than 90 degrees, and an obtuse angle has more than 90 degrees. The four right angles, formed when two perpendicular lines cross, add to up to how many degrees? $(300°)$ $(360°)$ $(400°)$

6. Perhaps you remember the name of the longest side, the diagonal side opposite the 90° angle in a right triangle. It's called the hypotenuse. What do you think the two sides next to the right angle are called? (legs) (arms) (tails)

7. Yes, the legs of the right triangle. Usually we designate the sides of a right triangle with lower-case letters, those corresponding to upper-case letters indicating the angle opposite that side. This means that if capital A and capital B are the two acute angles of a right triangle, small a and b are the two legs, or shorter sides. Capital C would then be the right angle. What would lower-case c designate? (the right angle) (a leg) (the hypotenuse)

8. Right. If you have studied program Mg 7 recently, you will recall the Pythagorean Theorem, which states that the length of the hypotenuse is the square root of "a" squared plus "b" squared. Another way to put this is to say that "the square of the hypotenuse equals the sum

of the squares of the legs."

9. For the purpose of many statements and examples in trigonometry, we'll consider a standard right triangle with angles at A, B, and C. The right side is vertical and the base is horizontal, so C is a right angle. What do we know about angle A? (it's always 45°) (it's always less than B) (it's always less than 90°)

10. Right. If we draw another vertical line, say D to E, we'd make another included triangle. There is a proportion, or equality of ratios, between the ratio of the line length AC divided by AB, and the ratio AE over AD. This proportion always occurs in similar triangles. What are the triangles that have the same angle A, and each have a 90° angle? (similar) (different)

11. Of course. This constant ratio of the base, or leg adjacent to angle A, divided by the hypotenuse, is called the "cosine" of the angle. Sometimes the word "cosine" is abbreviated as "cos" in lower case. How would you describe the cosine of angle A? (ratio of length of adjacent leg to the hypotenuse) (ratio of the angular size of A and C)

12. Yes. Another trigonometric ratio is the "sine," which is the ratio of the length of the leg in the reference triangle opposite to the reference angle A, to the length of the hypotenuse. Notice that this word "sine" is spelled differently from the ordinary word "sign." Sometimes it is abbreviated "sin," but there's nothing sinful about it. It's just a ratio.

13. Which of these is the ratio of the opposite leg over the hypotenuse? (sign) (sine) (cosine)

14. A third important ratio is the "tangent." This is the ratio of the length of the leg opposite the angle referred to, divided by the leg adjacent to the angle. Which of these ratios is a tangent? $(\frac{AC}{AB})$ $(\frac{BC}{AB})$ $(\frac{BC}{AC})$

15. Now we have the three important ratios which are the heart of trigonometry: sine, cosine and tangent. There are three more ratios which are just the inverse, or reciprocal, of the ratios. These are called, respectively, secant, cosecant, and cotangent, but we won't try to memorize them. You should memorize sine, cosine and tangent, however, so we'll take 30 seconds to drill.

16. Sine, cosine and tangent are "functions" of a selected angle. They may be considered for our drill as sides of a standard reference triangle with the selected angle at the left, on the base, and the right angle at the right. Which of these is the ratio of the opposite side over the hypotenuse? (sin) (cos) (tan)

17. Yes. Which is the opposite leg divided by the adjacent leg? (sin) (cos) (tan)

18. Right. Which is the opposite side over the hypotenuse? (sin) (cos) (tan)

19. Which is the adjacent side over the hypotenuse? (sin) (cos) (tan)

20. Correct. What ratio of side lengths is the tangent of an angle? $(\frac{opp.}{adjac.})$ $(\frac{adjac.}{hypot.})$ $(\frac{opp.}{hypot.})$

21. Yes. Now, what is a cosine? $(\frac{opp.}{adjac.})$ $(\frac{adjac.}{hypot.})$ $(\frac{opp.}{hypot.})$

22. Yes, and what ratio of side lengths is the sine of an angle? $(\frac{opp.}{adjac.})$ $(\frac{adjac.}{hypot.})$ $(\frac{opp.}{hypot.})$

23. Right. You could repeat this sequence a few times, if you need to practice some more. We have been looking at an angle which increases counterclockwise from the horizontal to the right. On this protractor, you will notice the outside scale increasing in this way. The inside scale, however, starts on the left and increases clockwise. The angle and the ratios remain the same in either direction. To measure an angle, place the angle's vertex at the protractor's center mark, one leg along the lower edge, and read the angle mark where the other leg crosses the scale.

24. Here is part of a table of trigonometric values. Notice that when the angle is zero, the length of the opposite side is zero; and the two ratios, sine and tangent, which use the opposite side in the numerator, will of course become zero. Also, when the angle is zero, the adjacent side which is the numerator for the cosine, will be the same length as the hypotenuse, so the cosine value is one. What is the value of the sine of 30 degrees? (.300) (.360) (.500 or ½)

25. Yes, the sine of a 30-degree angle is ½, as you can see in this table. When you study trigonometry at length, it will help to memorize several ratio values, including the values at 0, 30, 45, 60, and 90 degrees. The sine goes from zero, to ½, to .707, to .866 to 1 for these five values. It is interesting to note that the values for the sine of 45°, and 60°, (.707 and .866 respectively), are the square roots of ½ and ¾. This is the result of the relationship noted in the Pythagorean Theorem. What does it say about sides a, b, and c? (They are all equal.) (They total 180°.) $(c^2 = a^2 + b^2)$

26. There are a large number of uses for these trigonometric functions, or ratios. Many physical relationships can be surveyed, or measured indirectly, by using them. In recording horizontal angles, for surveys and the like, it is often standard to measure clockwise from north or other reference direction. In measuring vertical angles, the horizontal at that point on the earth is usually used.

27. Vertical angles are often expressed as the "angle of elevation." You could sight along a protractor, or use a transit, or a sextant, to measure the angle from the horizontal to the sun or a star, or the top of a building or pole. If you were on an elevated spot, and depressed your sighting instrument below the horizontal to measure something below your level, what kind of angle do you suppose we would call it? (angle of superposition) (angle of depression)

28. We can solve some elevation problems by using the length of shadows to determine the height of buildings by setting up similar triangles. We just measure the shadow length of some vertical object whose height we knew, to get a ratio of vertical height to horizontal shadow length. If you measured the elevation angle, what function of it would involve these two legs of your triangle? (sine) (cosine) (tangent)

29. Right. At a given time the sun might be 45 degrees from the horizontal, or at an elevation angle of 45 degrees. This would give you a shadow the same length as the height of the object.

What is the tangent of 45 degrees? (.500) (.707) (1.000)

30. Right. But on a cloudy day, or if you couldn't wait until the shadow reached some handy ratio of a reference height, you might use a protractor or transit to measure the elevation angle, then multiply the measured horizontal distance to the object by the tangent of the angle to get its height. The tangent of 13 degrees is .225. What is the height of a building which is 1000 feet away when observed to have an elevation angle of 13 degrees? (225 ft. high) (1000 ft. high) (1300 ft. high)

31. Yes, and if you knew the building was 225 feet high, you could find the point 1000 feet away by moving out until the elevation angle is 13 degrees. Remember that the span between your thumb and middle finger, held at arm's length in front of you, is 14 or 15 degrees, with a tangent of about .250 or ¼. If you walked up the mall toward the U.S. Capitol until the distance between the Washington Monument base door and the top window, about 500 feet high, fills your span, how far away would you be? (500 ft.) (1000 ft.) (2000 ft.)

32. Right. This four-to-one tangent ratio can be quite useful. Another handy tangent ratio is based on the width of your thumb, which held in front of you is about 2 degrees wide, or covers a height or width about one thirtieth of the distance to an object. If you approached a city and noticed that your thumb width just covered the height of a 25-story building which was about 250 feet high, how far would you be from it? (750 feet) (250 × 30 = 7500 ft. or 1.4 miles) (7.5 miles)

33. Yes. Sometimes you may know, or need to know, a diagonal distance which would make up the hypotenuse of a right triangle. In this case, which ratio would you use? (tangent) (cotangent) (sine or cosine)

34. Right. If you were driving across western Kansas on Interstate 70, and noted that the Interstate was heading 30 degrees north of west, how far west, then how far north again would you have to travel to visit Oakley if the bypass rejoined the Interstate five miles further ahead? (4.33 mi. west, 2.5 mi. north) (8.66 mi. west, 7.07 mi. north)

35. Yes. If there is a crane with a boom 50 feet long, and a line extended 40 feet down, what ratio would you first think about to solve some problem using this information? (sine) (cosine) (tangent)

36. Yes. The sine, of course, is the ratio of the opposite side length to the hypotenuse of a right triangle. Sometimes, of course, you don't have a right triangle, but have to create one. This is easy when you have a symmetrical figure, such as an isosceles triangle.

37. The base of this A-frame cabin is easily measured. It is 20 feet wide, and the roof-sides slope 60 degrees from the floor. How long are the rafters? (17.33 ft) (20 ft) (26.66 ft)

38. Yes. You mentally dropped a vertical line from the roof ridge, which formed two right angles with base legs ten feet long. The cosine of 60 degrees is ½. Divide the base, ten feet, by the cosine and you get 20 feet for the hypotenuse.

Vector Operations

1. Alternating current electrical values are often considered as quantities with both magnitude and phase, or direction. A quantity which has both magnitude and direction is called a "vector." Which of these would you consider a vector? (a pound of beans) (a quart of oil) (a mile walk to the store)

2. Right. Quantities which can be expressed by just specifying the amount of their value in units, are called "scalar" quantities. What is the name of the kind of quantity which has both amount and direction? (scalene) (victor) (vector)

3. Right. Which of these are vector quantities? (a 12-inch movement; a 10-pound force) (5 gallons of gasoline; 2 quarts of milk) (a 10-minute speech; a 2-hour flight)

4. Yes, changes of position, called displacements, and applied forces, may have both amount and direction specified. The direction may be described in two dimensions, which is easy to display, or in three dimensions or more, which are somewhat harder to show by diagrams.

5. We will consider primarily two-dimensional vectors, and deal only with their addition and subtraction, since this will be sufficient for most electronics problems. We may add several vectors, however many, 5 or 10 or even 100, if they are all in the same two dimensions.

6. If a surveyor were seeking a pass through some mountains, he might go 2 miles north, then 3 miles northeast, then a mile east, then 5 miles north, and so on. He would need to be able to add up all of the traverses, and find out how far he had gone, and in what direction. What would this be called? (scalar summation) (vector addition)

7. Right. A guy wire is needed to brace a pole at the end of a cable line. The cable's pull is equal to the sum of the pole's support and the guy line's tension. What kind of sum is this? (algebraic sum) (vector sum)

8. Right. An algebraic sum is a one-dimensional total of amounts which may have both positive and negative values, but along only one line. A vector sum combines values which may be more than just positive or negative; they also have directions, which may vary in two or more dimensions.

9. It is easier to understand two-dimensional vectors when they are represented graphically by lines or arrows. The length of the arrow line represents the magnitude, or amount, of the vector, and the direction it is drawn represents its direction, or phase angle. We could show the forces on the cable, the guy wire and the pole by these three arrows. What would be the sum of these three forces? (zero) (1763 pounds)

10. If you stepped off 100 paces east, and then 100 paces north, how could you describe, to some-one else, how to walk the shortest way to the same position, from the same starting place? (walk 100 paces east, 100 paces north) (walk 141.4 paces northeast) (walk 100 paces north, 100 paces east)

11. Yes. If you had a compass, and went through the woods, walking 62 paces in a direction 45 degrees clockwise from north, 50 paces at 90 degrees, 38 paces at 120 degrees, then 58 paces at 70 degrees, how could you tell someone how to come to you the quickest way? (use vector addition) (tell them to follow your trail)

12. Yes. One way to add vectors, approximately, is to draw the arrows carefully to a scale, in proportion to the amount of each vector, and in their specified direction, with one vector begin-ning at the end of another, and so on, until they are all laid out in series. Then the sum is the distance and direction from the beginning of the first arrow to the end of the last one.

13. This graphical method, when used to add only two vectors is sometimes called the triangular vector addition. Since vectors, like forces, and even voltages, are often shown originating from the same reference origin, this graphical method of addition may be considered to create a parallelogram with two triangles by moving each of the two vectors to the end of the other vector.

14. Let's say you walked 40 paces, 60° clockwise from north. Then you walked 20 paces 30° clock-wise from north of this point. Using a protractor, you'd lay off, say 4 inches at 60° from north, and 2 inches at 30° from north. Draw a line from beginning to end. What is the approximate result in distance and degrees from north? (5 inches at 90°) (5 inches at 4°) (6 inches at 50°)

15. Yes. You may recall that there are two common plane vector systems, the rectangular, or xy, coordinate system, and the polar, or radius-angle system. Sometimes the polar coordinate system is called the R-theta system, where R is a radius and theta is the name of the angle. And unfortun-ately for compass readers, the theta angles are considered counterclockwise from a horizontal line to the right of the origin point.

16. Now, for a moment please forget the way we measure compass degrees from north. In what direction do we measure angles in the polar coordinate, R-theta system? (◿θ) (◺θ) (◹θ)

17. Yes. The accuracy of the result of a graphical addition depends upon the tools you use and the skill and care you employ. For a long time, mariners relied on vector addition of ship movement, and used sharp-pointed dividers, adjusted carefully to maps and charts at a large scale.

18. The method of adding more than two displacements or other vector quantities by this graphical method is called the polygon method, and the polygon drawn this way is completed and enclosed by drawing the vector sum of all the values as a diagonal line from the beginning of the first to the end of the last vector.

19. Vector subtraction is similar to algebraic subtraction, but you must remember that it also has

direction, as well as magnitude. For example, if an airplane flies at 200 miles per hour exactly into a headwind of 50 miles per hour, you might either add the minus-50 miles per hour wind algebraically or just subtract 50, which amounts to the same thing.

20. But if the 50 miles-per-hour wind is a crosswind to the airplane's heading, you must be careful to add or subtract in the proper direction. A vector subtraction, then, is the same as adding a vector with a direction directly opposite, or 180 degrees from, the vector which is being subtracted.

21. In electronics, we more often use numerical values, rather than graphical representations, and rectangular coordinates rather than polar coordinates. Any vector can be expressed in rectangular coordinates, as the sum of the two vectors parallel to the x axis and the y axis. These two rectangular coordinate elements of the vector have what relationship? (They are parallel) (x is always smaller than y.) (They are at right angles.)

22. Right. We can easily resolve any two-dimensional vector into x and y rectangular components by a graphical procedure. Just draw lines parallel to the x and y axis, and measure the distance from the intersection with these axes to the vector's origin point.

23. But if you have a slide rule or a calculator or a computer with the trigonometric functions, sine or cosine, you can immediately obtain the x and y components of any vector. Even using sine and cosine tables, and a multiplying calculator is not much effort, and will provide as much precision as you need.

24. For example, take vector R-bar. It has a radius magnitude of R, and an angle of direction, theta. From your knowledge of trigonometry, you know that its rectangular components are R times sine θ upwards, and R times cosine θ, to the right.

25. You recall that the sine of the angle is the side opposite the angle over the hypotenuse of the right triangle, which in this case is the radius vector R-bar. The side opposite is the "up" or "y" component of R.

26. What is the sine of angle theta? (the side opposite the angle, over the hypotenuse) (the side adjacent to the angle, over the hypotenuse)

27. Yes. What is the cosine of angle theta? (the side opposite the angle, over the hypotenuse) (the side adjacent to the angle, over the hypotenuse)

28. Right. You can reverse the process, and find the value of R and θ for the vector by combining the x and y components. You can get R by squaring x, squaring y and taking the square root of their sum. What relationship does this use? (Hypocratic Oath) (Socratic Syllogism) (Pythagorean Theorem)

29. Right. "The length of the hypotenuse of a right triangle equals the square root of the sum of the squares of the sides." You can perform this in practice with a slide rule or calculator, using the squaring and square-root procedures you have learned in earlier programs.

30. Now that you have x, y, and R, the hypotenuse, you can find the angle θ by dividing y by R. This gives you the sine of θ, so you look up the angle, θ, in the tables, on your slide rule, or use a fancy calculator. How else could you find the angle? ($\frac{x}{R} = \cos\theta$) ($\frac{y}{x} = \tan\theta$) (either way is OK)

31. You should remember the values of sine and cosine for 30 degrees, 45 degrees and 60 degrees. They're very easy. Sine and cosine of 30 degrees are .5 and .866, respectively. Sine and cosine of 45 degrees are both .707, which by the way is the square root of ½. Sine and cosine of 60 degrees are .866 and .5, respectively, the reverse of 30 degrees, naturally. And you should remember that sine and cosine of 0 degrees are zero and one, while sine and cosine of 90 degrees are one and zero! Better look these over for a moment.

32. What are the sine and cosine of 45°? (sin = .5, cos = .866) (sin = .707, cos = .707) (sin = .866, cos = .5)

33. What are the sine and cosine of 30°? (sin = .5, cos = .886) (sin = .707, cos = .707) (sin = .866, cos = .5)

34. Now suppose you have a vector of 10 units size, say 10 volts AC which is at a 30 degree angle clockwise from the x axis. Resolve it into x and y components. What are they? (y = 5 volts, x = 8.66 volts) (y = 7.07 volts, x = 7.07 volts) (y = 8.66 volts, x = 5 volts)

35. Yes. Now, let's combine some rectangular components of a vector. What if we had a vector with an x-component of 3 volts and a vertical y-component of 4 volts? What's the magnitude of the resultant vector? (4.66 volts) (5 volts) (5.33 volts)

36. Correct. And what is the angle of the vector? Let's see, the sin of the vector is y over R, which is 4 over 5, or .80. Now, estimate the angle. (42°) (53°) (64°)

37. Yes. The sine of 45° is .707, the sine of 60° is .866, so .80 would be between 45° and 60°. Of course, if you remember this 3-4-5 integral Pathagorean right triangle, you may have recalled that it has 37 and 53 degree angles in it.

38. What's the magnitude of the resultant vector with components at right angles of 5 and 12? (13) (16) (19)

39. What's the value of sin θ? ($\frac{y}{R}$) ($\frac{x}{y}$) ($\frac{x}{R}$)

40. In your studies of electronics, you will need to have a good understanding of vector quantities, so you should repeat this program two or three times and do some extra study. How could you describe a vector? (quantity with magnitude only) (quantity with magnitude and direction)